Math 314 Fall 2013 Homework 8

Due Wednesday Nov. 13 5pm in Assignment Box (CAB 3rd Floor)

- There are 6 problems, each 5 points. Total 30 points.
- Please justify all your answers through proof or counterexample.

Question 1. *Let* $f(x) = \exp[x \ln x]$. *Calculate* $f'(x)$.

Question 2. *Let* $f(x) = \arccos x$. *Calculate* $f'(x)$.

Question 3. *Is* $x_0 = \frac{1}{20\pi}$ *a local maximizer for* $f(x) = (1 + (\sin x)^4) \cos\left(\frac{1}{x}\right)$*? Justify your answer.*

Question 4. *Prove Cauchy's Mean Value Theorem.*

Question 5. *Let* $f(x) = e^x - 1 - \sin x$. *Prove that* $f(x) \geqslant 0$ *for all* $x \geqslant 0$.

Question 6. *Prove*

$$\forall x \in (-1/2, 1/2), \qquad 3\arccos x - \arccos(3x - 4x^3) = \pi. \tag{1}$$

You can use the result from Question 2.