MATH 314 FALL 2013 HOMEWORK 5

DUE WEDNESDAY OCT. 16 5PM IN ASSIGNMENT BOX (CAB 3RD FLOOR)

- There are 6 problems, each 5 points. Total 30 points.
- Please justify all your answers through proof or counterexample.

Question 1. *Let $\{x_n\} = \{x_1, x_2, ...\}$ be a **sequence**. Denote*

$$M := \limsup_{n \longrightarrow \infty} x_n, \qquad m := \liminf_{n \longrightarrow \infty} x_n. \tag{1}$$

Critique the following claim:

$$\forall n \in \mathbb{N}, \qquad m - 100 < x_n < M + 100. \tag{2}$$

If it is true provide a proof, otherwise give a counter-example.

Question 2. *Are the following series convergent or divergent? Justify your answers.*

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n!}}, \qquad \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1} + \sqrt{n}}. \tag{3}$$

Question 3. *Let $x \in \mathbb{R}$. Consider the infinite series*

$$\sum_{n=1}^{\infty} \frac{x^n}{n^{\sqrt{2}}}. \tag{4}$$

Prove that it is convergent when $|x| \leqslant 1$ and divergent when $|x| > 1$.

Question 4. *Calculate the following limits. Provide justification whenever needed.*

$$\lim_{x \longrightarrow 1} \frac{x^2 - 3x + 2}{x^2 - 4x + 3}, \qquad \lim_{x \longrightarrow \infty} \left(\sqrt[3]{x+5} - \sqrt[3]{x} \right). \tag{5}$$

Question 5. *Discuss the existence/non-existence of the following limits. If a limit exists find the limit and justify your calculation, otherwise provide a proof.*

$$\lim_{x \longrightarrow \infty} \exp\left[\sin x + 1\right], \qquad \lim_{x \longrightarrow \infty} \exp\left[\sin x - 3x\right] \tag{6}$$

Question 6. *Let $f, g: \mathbb{R} \mapsto \mathbb{R}$ be functions. Let $a \in \mathbb{R}$. Critique the following claim:*

If $\lim_{x \longrightarrow a} f(x) = b$ and $\lim_{x \longrightarrow b} g(x) = L$, then $\lim_{x \longrightarrow a} g(f(x)) = L$.

If it is true provide a proof, otherwise find a counter-example.