Math 118 Winter 2015 Midterm 1 Review

- Midterm 1 coverage:
  - Lectures 1 - 16 and the exercises therein.
  - Required sections in Dr. Bowman's book.
  - Homeworks 1 – 4.
  - The exercises below are to help you on the concepts and techniques. The exam problems may or may not look like these exercises/problems.
- Remember you can always check your answer by differentiation.

1. Exercises.

Exercise 1. Calculate the following integrals.

$$\int \frac{x^3}{x+1}\,\mathrm{d}x; \qquad \int \frac{(x+1)^2}{x^{1/3}}\,\mathrm{d}x; \qquad \int \frac{x^2}{1+x^2}\,\mathrm{d}x; \qquad \int \frac{e^{3x}+1}{e^x+1}\,\mathrm{d}x; \qquad \int e^x\left(1-\frac{e^{-x}}{\sqrt{x}}\right)\mathrm{d}x. \tag{1}$$

Exercise 2. Calculate the following integrals.

$$\int \frac{\mathrm{d}x}{\sqrt{3-4\,x^2}}; \qquad \int \frac{\mathrm{d}x}{3+4\,x^2}; \qquad \int \frac{\mathrm{d}x}{\sqrt{x-2}+\sqrt{x+2}}; \qquad \int \frac{\mathrm{d}x}{\sqrt{x\,(1-x)}}; \qquad \int x\,e^{-x^2}\,\mathrm{d}x. \tag{2}$$

Exercise 3. Calculate the following integrals.

$$\int \ln(2+x^2)\,\mathrm{d}x; \qquad \int \frac{x}{\cos^2 x}\,\mathrm{d}x; \qquad \int x\arctan x\,\mathrm{d}x; \qquad \int x^2\arcsin x\,\mathrm{d}x. \tag{3}$$

Exercise 4. Calculate the following integrals.

$$\int \frac{x^8}{x^2-x-2}\,\mathrm{d}x; \qquad \int \frac{x+1}{x^3+2\,x^2-x-2}\,\mathrm{d}x; \qquad \int \frac{\mathrm{d}x}{x^4+x^2+1}. \tag{4}$$

Exercise 5. Calculate the following integrals.

$$\int \tan x\cdot\sin^2 x\,\mathrm{d}x; \qquad \int \cos^4 x\cdot\sin^3 x\,\mathrm{d}x; \qquad \int \frac{\mathrm{d}x}{2\sin x-\cos x}. \tag{5}$$

Exercise 6. Calculate the following integrals.

$$\int \frac{\sqrt{x}}{\sqrt[4]{x^3+1}}\,\mathrm{d}x; \qquad \int \frac{\sqrt{2+3\,x}}{\sqrt{x-3}}\,\mathrm{d}x. \tag{6}$$

2. More exercises.

Exercise 7. Calculate the following integrals.

$$\int \frac{\mathrm{d}x}{e^x+e^{-x}}; \qquad \int \frac{\mathrm{d}x}{\sqrt{x}\cdot\sqrt{1+\sqrt{x}}}; \qquad \int \frac{\mathrm{d}x}{x\sqrt{x^2+1}}; \qquad \int \frac{\mathrm{d}x}{\sqrt{(x^2+1)^3}}; \qquad \int \frac{\sqrt{x^2-1}}{x}\,\mathrm{d}x. \tag{7}$$

Exercise 8. Calculate the following integrals.

$$\int \sin(\ln x)\,\mathrm{d}x; \qquad \int x\,e^x\cos x\,\mathrm{d}x; \qquad \int \frac{x^8}{\sqrt{1-x^2}}\,\mathrm{d}x; \qquad \int (x^3+1)\,(\ln x)^4\,\mathrm{d}x; \qquad \int \ln\!\left(x+\sqrt{x^2+1}\right)\mathrm{d}x; \tag{8}$$

Exercise 9. Calculate the following integrals.

$$\int \frac{\mathrm{d}x}{1+\sqrt{x}+\sqrt{x+1}}; \qquad \int \frac{\sqrt{x^2+2\,x+2}}{x}\,\mathrm{d}x. \tag{9}$$

Exercise 10. Let $f(x)$ be continuous on $[0,1]$. Prove

$$\int_0^\pi x\, f(\sin x)\,\mathrm{d}x = \frac{\pi}{2}\int_0^\pi f(\sin x)\,\mathrm{d}x \tag{10}$$

3. Problems.

Problem 1. Calculate the following integrals.

$$\int \sqrt{\tan x}\,\mathrm{d}x; \qquad \int \frac{\mathrm{d}x}{(1+x^n)^{1+\frac{1}{n}}}; \qquad \int \frac{x\,e^x}{(1+x)^2}\,\mathrm{d}x. \tag{11}$$

Problem 2. Let $a, b, \alpha, \beta \in \mathbb{R}$ and $R(x, y, z)$ be rational. Prove that $\int R\big(x, \sqrt{a\,x+b}, \sqrt{c\,x+d}\big)\,\mathrm{d}x$ can always be integrated. Then Calculate $\int \frac{1+\sqrt{x}}{1-\sqrt{1-x}}\,\mathrm{d}x$.

Problem 3. [1]Let $P(x) := x^n + \cdots$ be a polynomial with n distinct real roots $x_1, ..., x_n$. Prove Newton's formula:

$$\frac{x_1^k}{P'(x_1)} + \cdots + \frac{x_n^k}{P'(x_n)} = \begin{cases} 0 & k = 1, 2, ..., n-2 \\ 1 & k = n-1 \end{cases}. \tag{12}$$

(Hint: Partial fraction.)

Problem 4. Prove $\int_0^{\pi/2} \cos(n\,x)\cos^n x\,\mathrm{d}x = \frac{\pi}{2^{n+1}}$.

Problem 5. Let

$$I_n := n\int_1^{1+\frac{1}{n}} \sqrt{1+x^n}\,\mathrm{d}x. \tag{13}$$

Prove that $\lim_{n\to\infty} I_n$ exists and calculate this limit.

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1. Richard Courant & Fritz John, *Introduction to Calculus and Analysis Vol 1*, Interscience Publishers, 1965.