

# Math 117 Fall 2014 Midterm Exam 3

Nov. 21, 2014 10AM - 10:50AM. TOTAL 20+2 PTS

NAME:

ID#:

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- There are five questions.
- Please write clearly and show enough work.

1

2

3

4

5

Total

**Question 1. (5 pts)** Prove by  $\varepsilon - \delta$ :  $f(x) := \begin{cases} 2 & x > 0 \\ 1 & x \leq 0 \end{cases}$  is continuous at every  $a \neq 0$  but discontinuous at 0.

**Question 2. (5 pts)** Let  $f(x) := \begin{cases} x + x^2 \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ . Prove that  $f$  is differentiable everywhere on  $\mathbb{R}$  and calculate  $f'(x)$ .

**Question 3. (5 pts)** *Prove or disprove:  $\sum_{n=1}^{\infty} \tan \frac{1}{n^2}$  converges. (You can use the convergence/divergence of  $\sum_{n=1}^{\infty} \frac{1}{n^a}$  without justification)*

**Question 4. (5 pts)** *Prove that there are exactly two solutions for the equation  $x^2 + 1 = 2 \cos x$ .*

**Question 5. (Extra 2 pts)** Find a function  $f: \mathbb{R} \rightarrow \mathbb{R}$  such that  $f$  is differentiable everywhere,  $f'(0) > 0$ , but there is no  $\delta > 0$  such that  $f$  is increasing on  $(-\delta, \delta)$ . Justify.

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