Math 117 Fall 2014 Homework 6

Due Thursday Oct. 30 3pm in Assignment Box

Question 1. (5 pts) *Let* $\sum_{n=1}^{\infty} a_n$ *be an infinite series. Prove that*

$$\sum_{n=1}^{\infty} a_n \text{ converges} \Longrightarrow \lim_{n\to\infty} a_n = 0. \quad (1)$$

Question 2. (5 pts) *Let* $r, c \in \mathbb{R}$. *Prove that* $\sum_{n=1}^{\infty} c\, r^n$ *converges if and only if* $|r| < 1$. *(You can use the conclusion of Question 1).*

Question 3. (5 pts) *Let* $\sum_{n=1}^{\infty} a_n$ *be an infinite series. Prove: If there is* $b_n \geqslant 0$ *such that* $\sum_{n=1}^{\infty} b_n$ *converges and* $\forall n \in \mathbb{N}\, |a_n| \leqslant b_n$, *then* $\sum_{n=1}^{\infty} a_n$ *converges.*

Question 4. (5 pts) *Let* $\sum_{n=1}^{\infty} a_n$ *be an infinite series.*

a) (2 pts) *If* $\limsup_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$, *then* $\sum_{n=1}^{\infty} a_n$ *converges;*

b) (2 pts) *If* $\liminf_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1$, *then* $\sum_{n=1}^{\infty} a_n$ *diverges;*

c) (1 pt) *Find an infinite series satisfying* $\limsup_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1$ *and also* $\liminf_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$. *You don't need to justify your claims.*

(You can use the conclusions from Questions 1 – 3)