

MATH 117 FALL 2014 HOMEWORK 2

DUE THURSDAY SEPT. 18 3PM IN ASSIGNMENT BOX

QUESTION 1. (5 PTS)

- a) (2 PTS) Find two irrational numbers a, b such that both $a + b$ and $a \times b$ are rational.
- b) (3 PTS) Can you find two irrational numbers a, b such that both $a + b$ and $a - b$ are rational? If you answer yes, find two such numbers; If you answer no, prove that this is not possible.

QUESTION 2. (5 PTS) Prove that $\sqrt{5} + \sqrt{11}$ is irrational.

QUESTION 3. (5 PTS) Calculate (using a computing device if necessary)

$$E_n := 1 + 1 + \frac{1}{2!} + \cdots + \frac{1}{n!} \quad (1)$$

for $n = 3, 5, 7$. For each $n = 3, 5, 7$, find the **smallest** $m \in \mathbb{N}$ such that

$$\left(1 + \frac{1}{m}\right)^m \geq E_n. \quad (2)$$

QUESTION 4. (5 PTS)

- a) (3 PTS) Prove that $\left(1 + \frac{1}{n}\right)^{n+1}$ is decreasing. (Hint:¹)
- b) (2 PTS) Use the result in a) to prove that $\left(1 + \frac{1}{n}\right)^n$ has a upper bound. Note that even if you cannot prove a), you still can apply the result in a) to b).

1. Prove that $\left[\left(1 + \frac{1}{n}\right)^{n+1}\right]^{-1}$ is increasing.