MATH 117 FALL 2014 LECTURE 1 (SEPT. 3, 2014)

- Please read syllabus carefully!

- About Calculus.

  ○ Etymology: Latin *calculus* ("a pebble or stone used for counting"), diminutive of *calx* (limestone).[1]

  ○ Why is it invented: To

    1. be able to calculate accurately the trajectory of moving objects;

    2. be able to deal with all kinds of curves.

  In some sense, ancient algebra and geometry was unified by Rene Descartes to analytic geometry, and then "upgraded" by Isaac Newton and Gottfried Wilhelm Leibniz to calculus. The "upgrade" is mainly from finite to infinite.

  Example 1. Recall Zeno's paradox about the race between Achilles and the tortoise.

  > Achilles could never catch up with the tortoise since whenever Achilles reaches somewhere the tortoise has been, he still has farther to go. Thus Achilles must reach infinitely many points before catching up, he could never overtake the tortoise.

  This paradox is unsolvable in finite mathematics.

  Exercise 1. Try to solve the following "The Trojan Fly" problem[2]:

  > Achilles travels at 8 mph but the tortoise manages only 1 mph. So Achilles has given it a start. At the point where Achilles catches the tortoise he draws level with a fly which proceeds to fly back and forth between them at 20 mph. After another hour Achilles is 7 miles ahead of the tortoise, but where is the fly?[3]

  ○ For the history and meaning of calculus, see "Mathematics: Making the Invisible Visible" by Keith Devlin. It's a 5 lecture series available on youtube and itunes.

  ○ Calculus plays central role in modern mathematics.

    − Equations $\xrightarrow{\text{Calculus}}$ Differential and Integral equations;

    − Geometry $\xrightarrow{\text{Calculus}}$ Differential Geometry; Topology.

    − Counting $\xrightarrow{\text{Calculus}}$ Theoretical Computer Sciences.

  ○ The development of calculus itself:

  $$\text{Calculus of one variable} \xrightarrow[\text{"infinitely many variables"}]{\text{more than one variables}} \begin{array}{l} \text{Multivariate Calculus} \\ \text{Functional Analysis} \end{array}$$

  In particular, calculus of infinitely many variables is so hard that most math majors only learn to study linear functions ("infinite" upgrade of functions like $f(x) = 3\,x$).

---

1. See How did the Romans Calculate: http://www.pims.math.ca/~hoek/opinions/Romans/.

2. Take from *Paradoxes from A to Z* by Michael Clark.

3. Ans: Anywhere between Achilles and the tortoise. This can be proved by the "Squeeze Theorem" which we will discuss when studying limits.