

**Math 570, Mathematical Biology**  
 Winter 2012

**Assignment 5, due March 29, 2012, in class**

**Exercise 19:** (5)

In 1975 Maginu studied a specific Turing model with nonlinearities

$$\begin{aligned} f_1(u, v) &= -u^3 + 0.6u - v \\ f_2(u, v) &= u - v \end{aligned}$$

1. Study the steady states of the purely kinetic equation (without diffusion) and determine trace and determinant of the Jacobian. Is the system of activator-inhibitor type?
2. On an interval  $[0, l]$  with homogeneous Neumann boundary conditions, calculate the threshold value for  $D_v/D_u$  such that pattern formation is possible.
3. Choose  $D_u = 0.1$  and  $D_v = 10$ . Calculate the unstable models, depending on the interval length  $l$ .
4. Find the minimal length  $l^*$  such that the homogeneous solution is unstable for each  $l > l^*$ .
5. Calculate the unstable modes for  $l = 4\pi$  explicitly.

**Exercise 20:** (4)

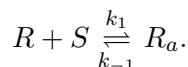
In class we studied a model for correlated random walk for  $(u^+, u^-)$  and we derived a chemotaxis equation. Thereby, the diffusion coefficient  $D$  and the chemotactic sensitivity  $\chi(S)$  were given as

$$D \approx \frac{\gamma^2}{\mu^+ + \mu^-}, \quad \chi(S)S_x \approx \frac{\gamma}{\mu^+ + \mu^-}(\gamma_x + \mu^+ - \mu^-).$$

In this exercise we want to calculate  $D$  and  $\chi$  explicitly. First we assume that

$$\gamma = \text{const.}, \quad \mu^\pm = \frac{\gamma}{2}(\gamma \mp \phi(S)_x).$$

The function  $\phi(S)$  describes the mechanism how a cell receives a signal and how it is transduced through the internal chemical pathway to the cell motor system. For simplicity we assume that the cell membrane has a fixed number  $R_0$  of signal receptors. Further we assume that binding of a signal molecule  $S$  to a receptor  $R$  activates the receptor  $R_a$  described by



We assume that this reaction is fast, relative to cell movement. Hence we study the quasisteady state approximation of this reaction. Finally, we assume that  $\phi(S)$  depends on the mean number of activated receptors

$$\phi(S) = \kappa_1 + \kappa_2 \bar{R}_a(S).$$

1. Calculate the mean number of active receptors,  $\bar{R}_a$ , from a quasisteady state approximation.
2. Calculate  $D$  and  $\chi(S)$  for this case.

**Exercise 21:** (5)

Consider the master equation for a random walk on a one-dimensional equidistant grid with transitional probabilities of the form

$$T_i^\pm = q(u_{i\pm 1})\phi(v_i)$$

- (a) Give a biological interpretation of these  $T_i^\pm$ .
- (b) Show that the continuous limit leads to a chemotaxis model of the form

$$u_t = (A(u, v)u_x - B(u, v)v_x)_x.$$

Find  $A(u, v)$  and  $B(u, v)$ .

**Exercise 22:** (6)

Consider a linear transport equation

$$p_t(t, x, v) + v \cdot \nabla p(t, x, v) = -\mu p(t, x, v) + \int T(v, v')p(t, x, v')dv'$$

for the special one-dimensional case of  $V = \{-s, +s\}$  and  $T(v, v') = c_1$ .

1. Find the constant  $c_1$  such that  $T$  is normalized.
2. Define  $a^\pm(t, x) = p(t, x, \pm s)$  and derive a system of equations for  $a^\pm$ . How is this system called?
3. Properly define the turning operator  $\mathcal{L}$  and find it's pseudo-inverse  $\mathcal{F}$ .
4. Find the parabolic limit.