University of Alberta
Mathematical and Statistical Sciences
Dr. Thomas Hillen
492-3395, thillen@ualberta.ca March 13, 2012

Math 570, Mathematical Biology
Winter 2012

Assignment 5, due March 29, 2012, in class

Exercise 19: (5)

In 1975 Maginu studied a specific Turing model with nonlinearities

$$\begin{aligned} f_1(u,v) &= -u^3 + 0.6u - v \\ f_2(u,v) &= u - v \end{aligned}$$

1. Study the steady states of the purely kinetic equation (without diffusion) and determine trace and determinant of the Jacobian. Is the system of activator-inhibitor type?
2. On an interval $[0,l]$ with homogeneous Neumann boundary conditions, calculate the threshold value for D_v/D_u such that pattern formation is possible.
3. Choose $D_u = 0.1$ and $D_v = 10$. Calculate the unstable models, depending on the interval length l.
4. Find the minimal length l^* such that the homogeneous solution is unstable for each $l > l^*$.
5. Calculate the unstable modes for $l = 4\pi$ explicitly.

Exercise 20: (4)

In class we studied a model for correlated random walk for (u^+, u^-) and we derived a chemotaxis equation. Thereby, the diffusion coefficient D and the chemotactic sensitivity $\chi(S)$ were given as

$$D \approx \frac{\gamma^2}{\mu^+ + \mu^-}, \qquad \chi(S)S_x \approx \frac{\gamma}{\mu^+ + \mu^-}(\gamma_x + \mu^+ - \mu^-).$$

In this exercise we want to calculate D and χ explicitly. First we assume that

$$\gamma = \text{const.}, \qquad \mu^{\pm} = \frac{\gamma}{2}(\gamma \mp \phi(S)_x).$$

The function $\phi(S)$ describes the mechanism how a cell recieves a signal and how it is transduced through the internal chemical pathway to the cell motor system. For simplicity we assume that the cell membrane has a fixed number R_0 of signal receptors. Further we assume that binding of a signal molecule S to a receptor R activates the receptor R_a described by

$$R + S \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} R_a.$$

We assume that this reaction is fast, relative to cell movement. Hence we study the quasisteady state approximation of this reaction. Finally, we assume that $\phi(S)$ depends on the mean number of activated receptors

$$\phi(S) = \kappa_1 + \kappa_2 \bar{R}_a(S).$$

1. Calculate the mean number of active receptors, $\bar{R}_a$, from a quasisteady state approximation.
2. Calculate D and $\chi(S)$ for this case.

Exercise 21: (5)

Consider the master equation for a random walk on a one-dimensional equidistant grid with transitional probabilities of the form

$$T_i^{\pm} = q(u_{i\pm1})\phi(v_i)$$

(a) Give a biological interpretation of these $T_i^{\pm}$.

(b) Show that the continuous limit leads to a chemotaxis model of the form

$$u_t = (A(u,v)u_x - B(u,v)v_x)_x.$$

Find $A(u,v)$ and $B(u,v)$.

Exercise 22: (6)

Consider a linear transport equation

$$p_t(t,x,v) + v \cdot \nabla p(t,x,v) = -\mu p(t,x,v) + \int T(v,v')p(t,x,v')dv'$$

for the special one-dimensional case of $V = \{-s, +s\}$ and $T(v,v') = c_1$.

1. Find the constant c_1 such that T is normalized.
2. Define $a^{\pm}(t,x) = p(t,x,\pm s)$ and derive a system of equations for $a^{\pm}$. How is this system called?
3. Properly define the turning operator $\mathcal{L}$ and find it's pseudo-inverse $\mathcal{F}$.
4. Find the parabolic limit.