University of Alberta
Mathematical and Statistical Sciences
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Math 570, Mathematical Biology
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Assignment 3, due March 01, 2012, in class

Exercise 11: (2)

Consider two types of tissues with α/β ratios of 3 Gy for healthy tissue and 10 Gy for tumour tissue. Let D denote the total radiation dose and d the dose per fraction. Assume that the healthy tissue can tolerate a biological effective dose of $BED_3 = B^*$. Assume the treatment is given to its limit of $BED_3 = B^*$. Show that the biologically effective dose to the tumour BED_{10} is monotonically decreasing in d and increasing in D. What does this imply for treatment planning?

Exercise 12: (4)

Calculate the Leah-Catchside protraction factor for brachytherapy. At time $t = 0$ radioactive isotopes are injected into the tumour. The initial dose intensity is D_0 and the half-life-time is $T = \ln 2/\mu$.

Exercise 13: (8)

Consider the pure birth process:

$$\frac{d}{dt}P_n(t) = b\Big((n-1)P_{(n-1)}(t) - nP_n(t)\Big).$$

1. Derive an equation for the moment generating function $g(s,t) = \sum_{n=0}^{\infty} s^n P_n(t)$. If initially we have n_0 cells, what are the initial conditions for g?
2. Solve the above initial value problem for g.
3. Calculate the extinction probability $p_0(t)$.
4. Calculate $g(1,t)$ and explain its meaning.
5. Calculate $\frac{\partial}{\partial s}g(1,t)$ and explain its meaning.

Exercise 14: (6)

We can also derive the diffusion equation from a birth-death process. To illustrate this we introduce a one-dimensional grid of gridsize h, and we denote the gridpoints as $x_i, i \in \mathbf{Z}$. Let $P_i(t)$ denote the number of particles at location x_i. Let $b > 0$ denote the probabilty of a jump to the right and $d > 0$ the probability of jump to the left. Note that $b + d < 1$.

1. Write down a master equation for the above described random walk process.
2. For the master equation in differential form (not time discrete) let the grid size $h \to 0$. Use Taylor expansion to derive a PDE for the particle density $p(t,x)$.
3. What happens for $b = d$, and what is the meaning of $b - d$ for $b \neq d$?