

Math 372, Mathematical Modelling
 Fall 2010

Assignment 3, due Nov 05, 12:00 in class.

Exercise 11:

[5]

A Montana farmer owns 45 acres of land. She is planning to plant each acre with wheat or corn. Each acre of wheat yields \$ 200 in profits, whereas each acre of corn yields \$ 300 in profits. For each acre of wheat the farmer needs 3 workers and 2 tons of fertilizer, and for each acre of corn she needs 2 workers and 4 tons of fertilizer. The farmer has 100 workers and 120 tons of fertilizer available. Determine how many acres of wheat and corn need to be planted to maximize profits. Use any method of solution you find appropriate. Interpret the results in plain language.

Exercise 12:

[5]

A local company restores cars and trucks for resale. Each vehicle must be processed in the refinishing/paint shop and the machine/body shop. On average, each car processed generates \$3000 profit, and each truck processed generates \$2000 profit. The refinishing/paint shop has 2400 work-hours available per month and the machine/body shop has 2500 work-hours available per month. A car requires 50 work-hours in the machine/body shop and 40 work-hours in the refinishing/paint shop, whereas a truck requires 50 work-hours in the machine/body shop and 60 work-hours in the refinishing/paint shop.

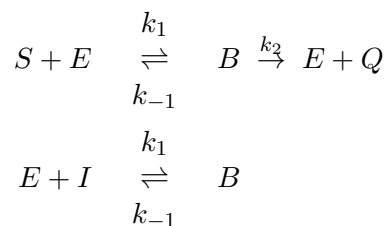
Determine a monthly production schedule that will maximize total profit. Interpret the results in plain language.

Exercise 13:

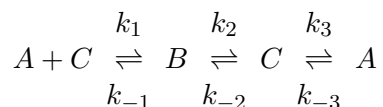
[4]

Describe the following chemical reactions in words and use the *law of mass action* to find appropriate differential equation models:

1.



2.



Exercise 14:

[6]

Now we are ready for a full analysis of the whale problem which we discussed early in class. Let $x(t)$ denote the number of blue whales and $y(t)$ the number of fin whales. They compete for the same resources and hence we use a competition model:

$$\begin{aligned}\frac{dx(t)}{dt} &= r_1x \left(1 - \frac{x}{K_1}\right) - \alpha_1xy \\ \frac{dy(t)}{dt} &= r_2y \left(1 - \frac{y}{K_2}\right) - \alpha_2xy\end{aligned}$$

with parameter values

$$r_1 = 0.5, r_2 = 0.8, K_1 = 150000, K_2 = 400000, \alpha_1 = 0.1, \alpha_2 = 0.2$$

Perform a full qualitative analysis:

1. Find all steady states.
2. Determine their stability through linearization.
3. Plot a phase portrait, null clines, and show typical solutions in the phase portrait.
4. Give a long-time prediction for the whale populations.