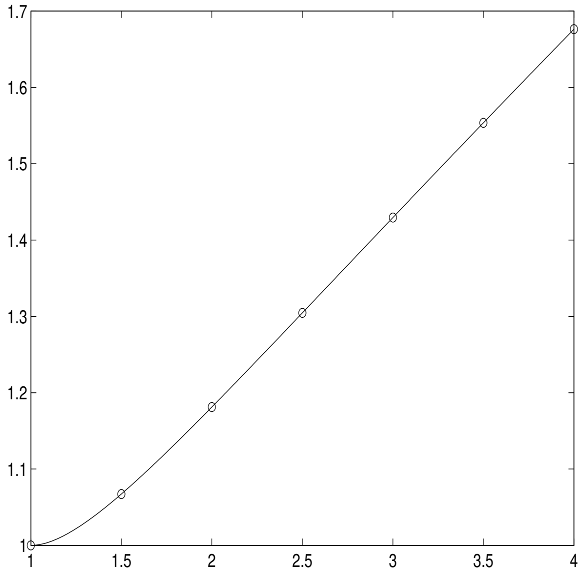
Assignment #4 Solutions

§**5.8 Problem 4a.** The following initial value problem is to be solved:

$$y' = \frac{y}{t} - \left(\frac{y}{t}\right)^2, \qquad 1 \le t \le 4, \quad y(1) = 1.$$

Use the extrapolation method with TOL=10e-6,hmax=0.5 and hmin=0.05. The function for this problem has been given in fnv4p4a.m

```
t0=1;tlast=4;w0=1;Fun='fnv4p4a';TOL=10e-6;hmax=0.5; hmin=0.05;
[T,W]=extra56(Fun,t0,tlast,w0,hmax,hmin,TOL);
t=1:.01:4;y=solv4p4a(t);
plot(T,W,'o',t,y);
format short e
[T,W,abs(W-solv4p4a(T))]
```



Plot for §5.8 Problem 4a.

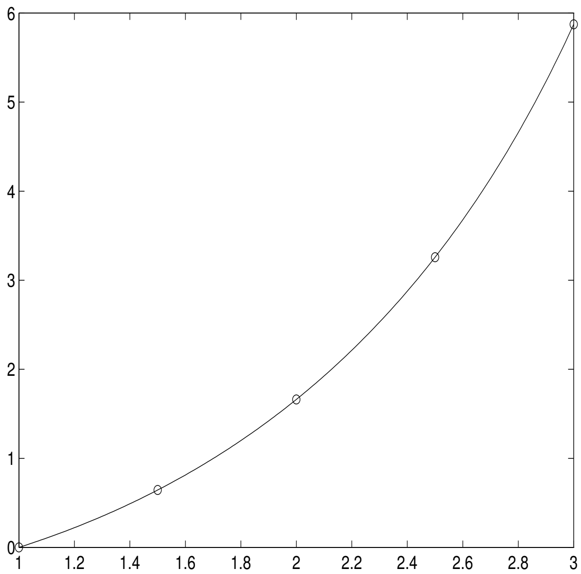
T	W	abs(Y-W)
1.0	1.0000e+00	0
1.5	1.0673e+00	5.3345e-07
2.0	1.1812e+00	2.8381e-06
2.5	1.3046e+00	2.1104e-06
3.0	1.4295e+00	2.0659e-06
3.5	1.5536e+00	2.1091e-06
4.0	1.6762e+00	2.1548e-06

§**5.8 Problem 4b.** The following initial value problem is to be solved:

$$y' = 1 + \frac{y}{t} + \left(\frac{y}{t}\right)^2, \qquad 1 \le t \le 3, \quad y(1) = 0.$$

Use the extrapolation method with TOL=10e-6,hmax=0.5 and hmin=0.05. The function for this problem has been given in fnv4p4b.m

```
t0=1;tlast=3;w0=0;Fun='fnv4p4b';TOL=10e-6;hmax=0.5; hmin=0.05;
[T,W]=extra56(Fun,t0,tlast,w0,hmax,hmin,TOL);
t=1:.01:3;y=solv4p4b(t);
```



Plot for §5.8 Problem 4b.

```
plot(T,W,'o',t,y);
format short e
[T,W,abs(W-solv4p4a(T))]
```

T	W	abs(Y-W)
1.0	0	1.0000e+00
1.5	6.4388e-01	4.2339e-01
2.0	1.6613e+00	4.8005e-01
2.5	3.2580e+00	1.9534e+00
3.0	5.8741e+00	4.4446e+00

§5.8 Problem 6. Here the given problem should be interpreted in the notation of our numerical methods. The problem is (substituting $P(t) = y(t)$ and putting in the numerical values for b and k):

$$y' = (2.9e-2) * y - (1.4e-7) * y^2, \qquad y(0) = 50976, \quad 0 \le t \le 5.$$

The goal is to find $P(5) = y(5)$. Now it is possible to solve this equation exactly, but we will use the extrapolation method to estimate the solution. You have the freedom to choose $hmax$, $hmin$ and TOL with the only restriction being that you obtain an answer and are reasonable sure this answer is close to being correct. Now the initial condition is given to 5 significant digits. Your choices of $hmax$, $hmin$ and TOL will of course make variations in your answers for the ones given here. First, make a function m-file fnv8p6.m to define the function $f(t, y)$:

```
function z=fnv8p6(t,y);
z=(2.9e-2)*y-(1.4e-7)*y.^2;
```

Now implement the extrapolation algorithm:

```
t0=1;tlast=5;w0=50976;Fun='fnv8p6';TOL=10e-6;hmax=0.1; hmin=0.001;
[T,W]=extra56(Fun,t0,tlast,w0,hmax,hmin,TOL);
plot(T,W,'o');
format short e
[T,W]
```

T	W
1.0	5.0976e+04
1.1	5.1088e+04
1.2	5.1199e+04
1.3	5.1311e+04
1.4	5.1423e+04

1.5	5.1535e+04
1.6	5.1648e+04
1.7	5.1760e+04
1.8	5.1873e+04
1.9	5.1986e+04
2.0	5.2099e+04
2.1	5.2212e+04
2.2	5.2325e+04
2.3	5.2439e+04
2.4	5.2552e+04
2.5	5.2666e+04
2.6	5.2780e+04
2.7	5.2894e+04
2.8	5.3009e+04
2.9	5.3123e+04
3.0	5.3238e+04
3.1	5.3352e+04
3.2	5.3467e+04
3.3	5.3583e+04
3.4	5.3698e+04
3.5	5.3813e+04
3.6	5.3929e+04
3.7	5.4045e+04
3.8	5.4161e+04
3.9	5.4277e+04
4.0	5.4393e+04
4.1	5.4509e+04
4.2	5.4626e+04
4.3	5.4743e+04
4.4	5.4859e+04
4.5	5.4976e+04
4.6	5.5094e+04
4.7	5.5211e+04
4.8	5.5329e+04
4.9	5.5446e+04
5.0	5.5564e+04

Only the last entry is significant. Since the population should be an integer, we find that the answer will be 55,564. NOTE: This answer is obtained even with a step size of $hmax = .5$.

§**5.9 Problem 1c.** The m-file that describes the system of equations to be solved could look like:

```
function z=fnv9p1c(t,U);
z=[-4*U(1)-2*U(2)+cos(t)+4*sin(t),3*U(1)+U(2) -3*sin(t)];
```

while the m-file of the actual solution would look like:

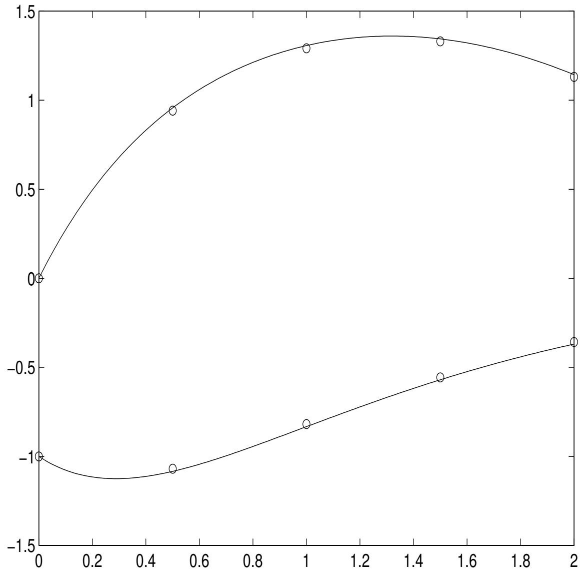
```
function z=sov9p1c(t);
z=[2*exp(-t)-2*exp(-2*t)+sin(t),-3*exp(-t)+2*exp(-2*t)];
```

Now apply the Runge-Kutta order 4 method to the system with $h = .5$ on the interval $0 \leq t \leq 2$ with the initial conditions $w0 = [0, -1]$.

```
yprime='fnv9p1c';w0=[0,-1];t0=0;tlast=2;N=4;
[T,W]=rksys57(yprime,t0,tlast,w0,N);
t=[0:.01:2]';u=sov9p1c(t);U=sov9p1c(T);
plot(T,W,'o',t,u)
[T,W,abs(W-U)]
```

T	W(1)	W(2)	abs(W(1)-U(1))	abs(W(2)-U(2))
0	0	-1.0000e+00	0	0

0.5	9.4135e-01	-1.0688e+00	1.5377e-02	1.5024e-02
1.0	1.2913e+00	-8.1833e-01	1.5219e-02	1.4635e-02
1.5	1.3304e+00	-5.5677e-01	1.3765e-02	1.3042e-02
2.0	1.1310e+00	-3.5783e-01	1.2306e-02	1.1546e-02



Plot for §5.9 Problem 1c.

§5.9 Problem 1f. The m-file that describes the system of equations to be solved could look like:

```
function z=fnv9p1f(t,U);
z=[4*U(1)-U(2)+U(3)-(t+2).^2,-U(1)+3*U(2)-2*U(3)+2*t.^2+t+15, U(1)-2*U(2)+3*U(3)-3*t.^2+t-10];
```

while the m-file of the actual solution would look like:

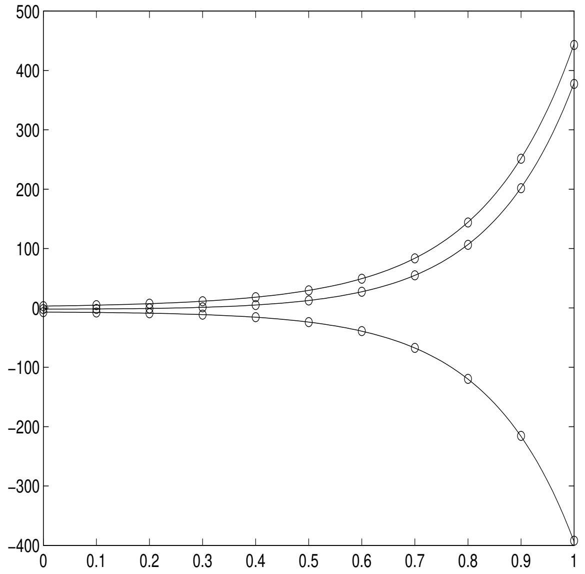
```
function z=sov9p1f(t);
z=[exp(6*t)+2*exp(3*t)+t,-exp(6*t)+exp(3*t)-2*exp(t)-5, exp(6*t)-exp(3*t)-2*exp(t)+t.^2];
```

Now apply the Runge-Kutta order 4 method to the system with $h = .1$ on the interval $0 \le t \le 1$ with the initial conditions $w0 = [3, -7, -2]$.

```
yprime='fnv9p1f';w0=[3,-7,-2];t0=0;tlast=1;N=10;
[T,W]=rksys57(yprime,t0,tlast,w0,N);
t=[0:.01:1]';u=sov9p1f(t);U=sov9p1f(T);
plot(T,W,'o',t,u)
[T,W]
[T,abs(W-U)]
```

T	W(1)	W(2)	W(3)
0	3.0000e+00	-7.0000e+00	-2.0000e+00
.1	4.6211e+00	-7.6819e+00	-1.7288e+00
.2	7.1616e+00	-8.9382e+00	-9.0741e-01
.3	1.1261e+01	-1.1283e+01	9.7319e-01
.4	1.8045e+01	-1.5669e+01	4.8620e+00
.5	2.9508e+01	-2.3862e+01	1.2517e+01
.6	4.9209e+01	-3.9106e+01	2.7178e+01
.7	8.3532e+01	-6.7363e+01	5.4798e+01
.8	1.4397e+02	-1.1955e+02	1.0629e+02
.9	2.5127e+02	-2.1566e+02	2.0163e+02
1	4.4300e+02	-3.9219e+02	3.7731e+02

T	abs(U(1)-W(1))	abs(U(2)-W(2))	abs(U(3)-W(3))



Plot for §5.9 Problem 1f.

0	0	0	0
.1	7.7454e-04	7.1350e-04	7.1336e-04
.2	2.7719e-03	2.6061e-03	2.6058e-03
.3	7.4739e-03	7.1365e-03	7.1359e-03
.4	1.7973e-02	1.7363e-02	1.7363e-02
.5	4.0623e-02	3.9590e-02	3.9589e-02
.6	8.8307e-02	8.6630e-02	8.6628e-02
.7	1.8689e-01	1.8424e-01	1.8424e-01
.8	3.8787e-01	3.8377e-01	3.8377e-01
.9	7.9300e-01	7.8677e-01	7.8677e-01
1	1.6022e+00	1.5929e+00	1.5928e+00

§5.9 **Problem 2c.** The m-file that describes the system of equations to be solved could look like:

```
function z=fnv9p2c(t,U);
z=[U(2),t.*log(t)-2*U(1)./(t.^2)+2*U(2)./t];
```

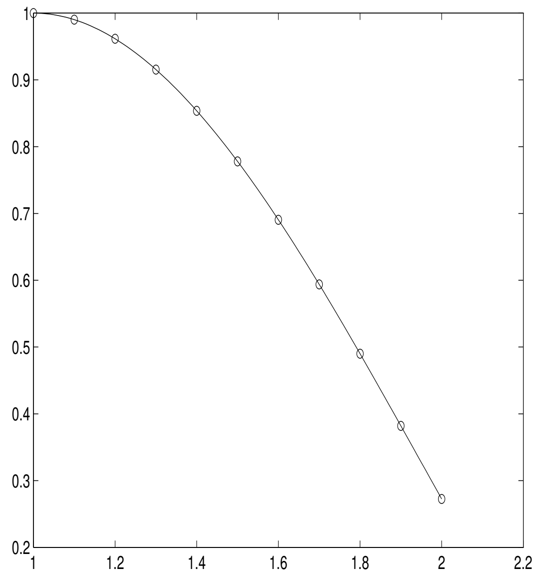
while the m-file of the actual solution would look like:

```
function z=sov9p2c(t);
z=(7/4)*t+(1/2)*t.^3.*log(t)-(3/4)*t.^3;
```

Now apply the Runge-Kutta order 4 method to the system with $h = .1$ on the interval $1 \leq t \leq 2$ with the initial conditions $w0 = [1, 0]$.

```
yprime='fnv9p2c';w0=[1,0];t0=1;tlast=2;N=10;
[T,W]=rksys57(yprime,t0,tlast,w0,N);
t=[1:.01:2]';u=sov9p2c(t);U=sov9p2c(T);
plot(T,W(:,1),'o',t,u)
[T,W(:,1), abs(W(:,1)-U)]
```

T	W(1)	abs(W(1)-U)
1.0	1.0000e+00	0
1.1	9.9018e-01	7.4726e-07
1.2	9.6152e-01	1.4532e-06
1.3	9.1546e-01	2.1266e-06
1.4	8.5364e-01	2.7742e-06
1.5	7.7797e-01	3.4014e-06
1.6	6.9056e-01	4.0122e-06
1.7	5.9373e-01	4.6099e-06



Plot for §5.9 Problem 2c.

1.8	4.8998e-01	5.1973e-06
1.9	3.8198e-01	5.7764e-06
2.0	2.7258e-01	6.3491e-06

§**5.9 Problem 2f.** The m-file that describes the system of equations to be solved could look like:

```
function z=fnv9p2f(t,U);
z=[U(2),U(3),exp(t)+2*U(1)+U(2)-2*U(3)];
```

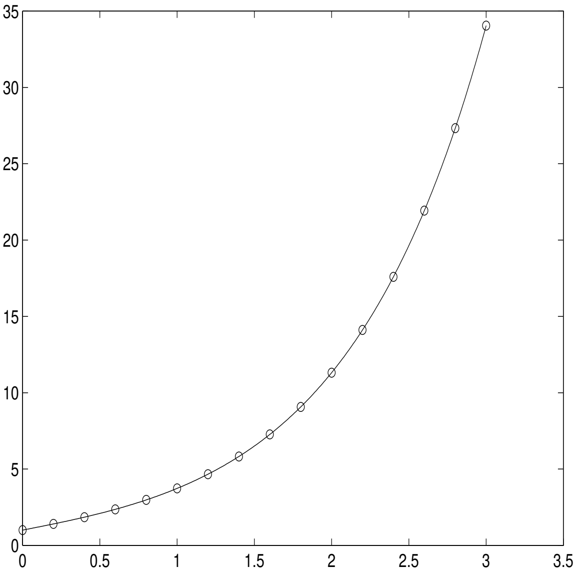
while the m-file of the actual solution would look like:

```
function z=sov9p2f(t);
z=(43/36)*exp(t)+(1/4)*exp(-t)-(4/9)*exp(-2*t)+ (1/6)*t.*exp(t);
```

Now apply the Runge-Kutta order 4 method to the system with $h = .2$ on the interval $0 \le t \le 3$ with the initial conditions $w0 = [1, 2, 0]$.

```
yprime='fnv9p2f';w0=[1,2,0];t0=0;tlast=3;N=15;
[T,W]=rksys57(yprime,t0,tlast,w0,N);
t=[0:.01:3]';u=sov9p2f(t);U=sov9p2f(T);
plot(T,W(:,1),'o',t,u)
[T,W(:,1), abs(W(:,1)-U)]
```

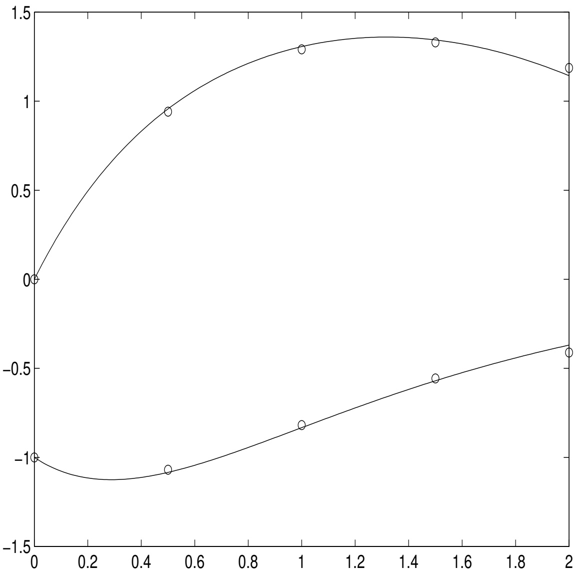
T	W(1)	abs(W(1)-U)
0	1.0000e+00	0
2.0000e-01	1.4063e+00	3.7051e-05
4.0000e-01	1.8492e+00	5.3497e-05
6.0000e-01	2.3619e+00	6.1340e-05
8.0000e-01	2.9776e+00	6.7811e-05
1.0000e+00	3.7316e+00	7.7492e-05
1.2000e+00	4.6646e+00	9.3657e-05
1.4000e+00	5.8244e+00	1.1912e-04
1.6000e+00	7.2691e+00	1.5679e-04
1.8000e+00	9.0698e+00	2.1014e-04
2.0000e+00	1.1314e+01	2.8351e-04
2.2000e+00	1.4111e+01	3.8250e-04
2.4000e+00	1.7594e+01	5.1435e-04
2.6000e+00	2.1931e+01	6.8840e-04
2.8000e+00	2.7329e+01	9.1670e-04
3.0000e+00	3.4044e+01	1.2147e-03



Plot for §5.9 Problem 2f.

§5.9 Problem 4c. Do Problem 1c with pcsys54.m. See the solution to Problem 1c for the setup. Apply the Predictor-Corrector of order 4 to the system with $h = .5$ on the interval $0 \leq t \leq 2$ with the initial conditions $w0 = [0, -1]$.

```
yprime='fnv9p1c';w0=[0,-1];t0=0;tlast=2;N=4;
[T,W]=pcsys54(yprime,t0,tlast,w0,N);
t=[0:.01:2]';u=sov9p1c(t);U=sov9p1c(T);
plot(T,W,'o',t,u)
[T,W,abs(W-U)]
```



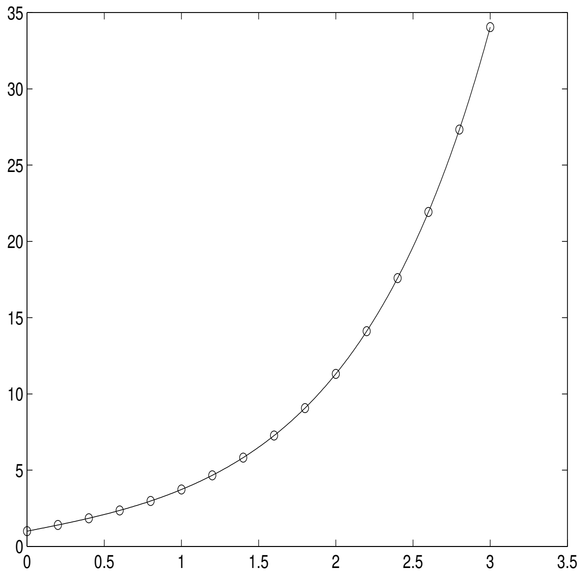
Plot for §5.9 Problem 4c.

T	W(1)	W(2)	abs(W(1)-U(1))	abs(W(2)-U(2))
0	0	-1.0000e+00	0	0
.5	9.4135e-01	-1.0688e+00	1.5377e-02	1.5024e-02
1.0	1.2913e+00	-8.1833e-01	1.5219e-02	1.4635e-02

1.5	1.3304e+00	-5.5677e-01	1.3765e-02	1.3042e-02
2.0	1.1866e+00	-4.1078e-01	4.3275e-02	4.1403e-02

§5.9 **Problem 5f.** Do Problem 2f with pcsys54.m. See the setup in Problem 2f. Apply the Predictor-Corrector of order 4 method to the system with $h = .2$ on the interval $0 \le t \le 3$ with the initial conditions $w0 = [1, 2, 0]$.

```
yprime='fnv9p2f';w0=[1,2,0];t0=0;tlast=3;N=15;
[T,W]=pcsys54(yprime,t0,tlast,w0,N);
t=[0:.01:3]';u=sov9p2f(t);U=sov9p2f(T);
plot(T,W(:,1),'o',t,u)
[T,W(:,1), abs(W(:,1)-U)]
```



Plot for §5.9 Problem 5f.

T	W(1)	abs(W(1)-U)
0	1.0000e+00	0
2.0000e-01	1.4063e+00	3.7051e-05
4.0000e-01	1.8492e+00	5.3497e-05
6.0000e-01	2.3619e+00	6.1340e-05
8.0000e-01	2.9777e+00	1.0033e-04
1.0000e+00	3.7319e+00	1.5893e-04
1.2000e+00	4.6649e+00	1.5509e-04
1.4000e+00	5.8247e+00	1.4107e-04
1.6000e+00	7.2694e+00	1.2184e-04
1.8000e+00	9.0701e+00	1.0476e-04
2.0000e+00	1.1315e+01	9.6705e-05
2.2000e+00	1.4111e+01	1.0098e-04
2.4000e+00	1.7595e+01	1.2052e-04
2.6000e+00	2.1932e+01	1.5883e-04
2.8000e+00	2.7330e+01	2.2031e-04
3.0000e+00	3.4045e+01	3.1077e-04