Assignment #3 Solutions

The following initial value problem is used in problems 3a, 5a, and 6a of section 5.6 and problem 4a of section 5.7:

$$y' = \frac{y}{t} - \left(\frac{y}{t}\right)^2, \qquad 1 \le t \le 2, \quad y(1) = 1.$$

(See Solutions to Assignment 2 for a description of the MATLAB function file.)

The initial value problem in problems 3b, 5b, and 6b of section 5.6 and problem 4b of section 5.7 is

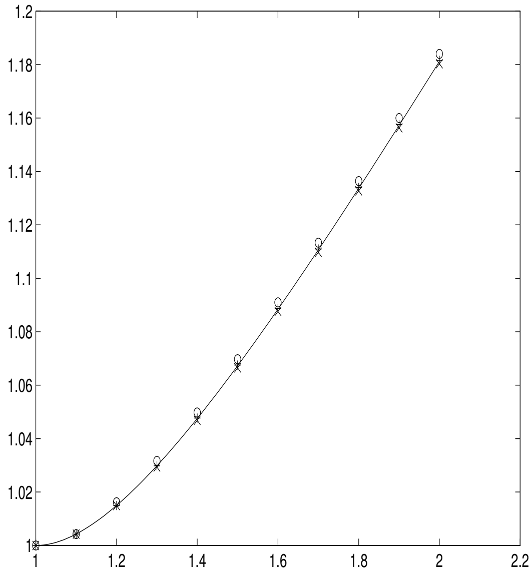
$$y' = 1 + \frac{y}{t} + \left(\frac{y}{t}\right)^2, \qquad 1 \le t \le 3, \quad y(1) = 0.$$

(Again see Solutions to Assignment 2 for a description of the MATLAB function file.)

§5.6 Problem 3a. This problem requires the application of the Adams-Bashforth methods with 2, 3 and 4 steps and step size $h = 0.1$. We present the MATLAB calls (using function files as described in the solutions to Assignment 2) and plot the various output against the actual solution (the program listing below will show what symbols are used for each method).

```
t0=1;tlast=2;w0=1;Fun='fnv4p4a';N=10;
[T,W2]=ab_2step(Fun,t0,tlast,w0,N);
[T,W3]=ab_3step(Fun,t0,tlast,w0,N);
[T,W4]=ab_4step(Fun,t0,tlast,w0,N);
t=1:.01:2;y=solv4p4a(t);Y=solv4p4a(T);
format short e
[T,W2,abs(Y-W2),W3,abs(Y-W3),W4,abs(Y-W4)]
plot(T,W2,'o',T,W3,'x',T,W4,'+',t,y);
```

T	W2	abs(Y-W2)	W3	abs(Y-W3)	W4	abs(Y-W4)
1.0	1.0000e+00	0	1.0000e+00	0	1.0000e+00	0
1.1	1.0043e+00	2.2415e-07	1.0043e+00	2.2415e-07	1.0043e+00	2.2415e-07
1.2	1.0162e+00	1.2459e-03	1.0150e+00	3.1076e-07	1.0150e+00	3.1076e-07
1.3	1.0317e+00	1.8685e-03	1.0294e+00	4.5584e-04	1.0298e+00	3.4637e-07
1.4	1.0498e+00	2.2326e-03	1.0469e+00	6.6088e-04	1.0477e+00	1.9391e-04
1.5	1.0697e+00	2.4517e-03	1.0665e+00	7.8357e-04	1.0675e+00	2.7386e-04
1.6	1.0910e+00	2.5877e-03	1.0876e+00	8.4899e-04	1.0888e+00	3.2397e-04
1.7	1.1133e+00	2.6744e-03	1.1098e+00	8.8592e-04	1.1110e+00	3.4434e-04
1.8	1.1364e+00	2.7310e-03	1.1327e+00	9.0701e-04	1.1340e+00	3.5577e-04
1.9	1.1600e+00	2.7689e-03	1.1563e+00	9.1928e-04	1.1576e+00	3.6145e-04
2.0	1.1840e+00	2.7950e-03	1.1803e+00	9.2654e-04	1.1816e+00	3.6446e-04

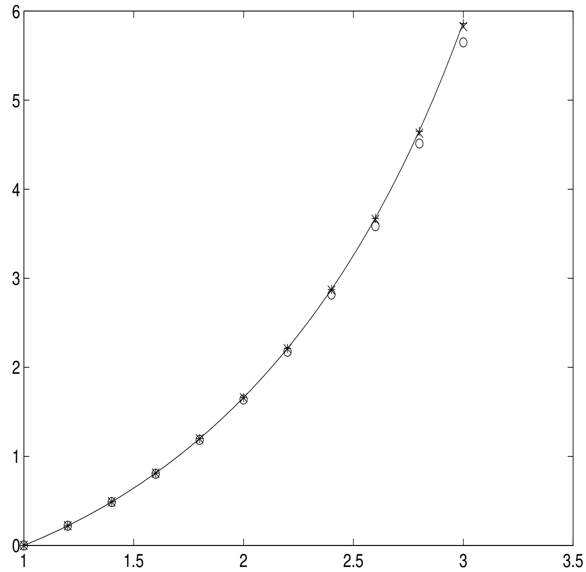


Plot for §5.6 Problem 3a.

§5.6 Problem 3b. Use Heun's Method on the problem in 4b.

```
t0=1;tlast=3;w0=0;Fun='fnv4p4b';N=10;
[T,W2]=ab_2step(Fun,t0,tlast,w0,N);
[T,W3]=ab_3step(Fun,t0,tlast,w0,N);
[T,W4]=ab_4step(Fun,t0,tlast,w0,N);
t=1:.01:3;y=solv4p4b(t);Y=solv4p4b(T);
format short e
[T,W2,abs(Y-W2),W3,abs(Y-W3),W4,abs(Y-W4)]
plot(T,W2,'o',T,W3,'x',T,W4,'+',t,y);
```

T	W2	abs(Y-W2)	W3	abs(Y-W3)	W4	abs(Y-W4)
1.0	0	0	0	0	0	0
1.2	2.2125e-01	2.9345e-06	2.2125e-01	2.9345e-06	2.2125e-01	2.9345e-06
1.4	4.8675e-01	2.9267e-03	4.8968e-01	2.5027e-06	4.8968e-01	2.5027e-06
1.6	8.0549e-01	7.2647e-03	8.1243e-01	3.2103e-04	8.1275e-01	5.7854e-07
1.8	1.1857e+00	1.3746e-02	1.1982e+00	1.2277e-03	1.1990e+00	3.9643e-04
2.0	1.6378e+00	2.3487e-02	1.6584e+00	2.8504e-03	1.6603e+00	9.7576e-04
2.2	2.1754e+00	3.8123e-02	2.2080e+00	5.5031e-03	2.2117e+00	1.7570e-03
2.4	2.8164e+00	6.0157e-02	2.8668e+00	9.7842e-03	2.8735e+00	3.0194e-03
2.6	3.5849e+00	9.3557e-02	3.6617e+00	1.6727e-02	3.6733e+00	5.1487e-03
2.8	4.5138e+00	1.4482e-01	4.6305e+00	2.8138e-02	4.6499e+00	8.7714e-03
3.0	5.6491e+00	2.2498e-01	5.8268e+00	4.7299e-02	5.8590e+00	1.5106e-02



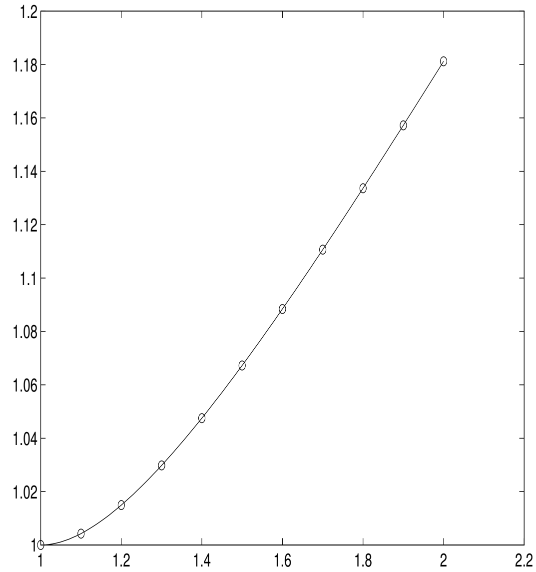
Plot for §5.6 Problem 3b.

§5.6 Problem 5a. Use the Predictor-Corrector Algorithm 5.4 on problem 3a.

```
t0=1;tlast=2;w0=1;Fun='fnv4p4a';N=10;
[T,W]=precor54(Fun,t0,tlast,w0,N);
t=1:.01:2;y=solv4p4a(t);Y=solv4p4a(T);
plot(T,W,'o',t,y);[T,W,abs(W-Y)]
```

T	W	abs(W-Y)
1.0	1.0000e+00	0
1.1	1.0043e+00	2.2415e-07
1.2	1.0150e+00	3.1076e-07
1.3	1.0298e+00	3.4637e-07
1.4	1.0475e+00	1.1192e-05
1.5	1.0672e+00	1.6134e-05
1.6	1.0884e+00	1.8583e-05
1.7	1.1106e+00	1.9833e-05
1.8	1.1336e+00	2.0484e-05

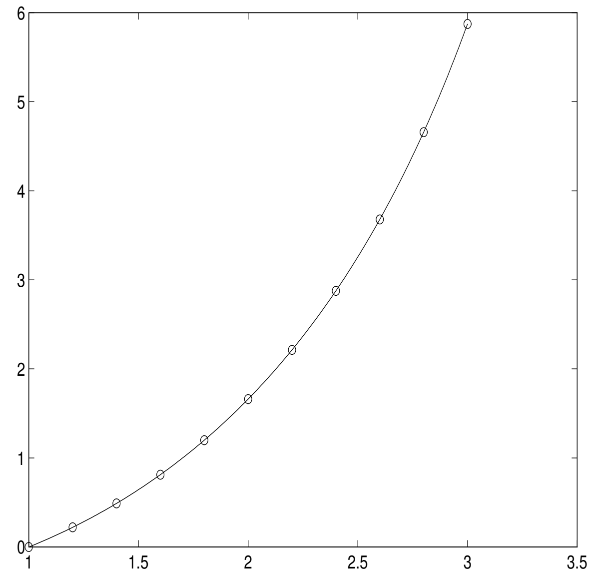
1.9	1.1572e+00	2.0825e-05
2.0	1.1812e+00	2.1002e-05



Plot for §5.6 Problem 5a.

§5.6 Problem 5b. Predictor-Corrector 5.4 applied to problem 3b.

```
t0=1;tlast=3;w0=0;Fun=´fnv4p4b´;N=10;
[T,W]=precor54(Fun,t0,tlast,w0,N);
t=1:.01:3;y=solv4p4b(t);Y=solv4p4b(T);
plot(T,W,´o´,t,y);[T,W,abs(W-Y)]
```



Plot for §5.6 Problem 5b.

T	W	abs(W-Y)
1.0	0	0
1.2	2.2125e-01	2.9345e-06
1.4	4.8968e-01	2.5027e-06
1.6	8.1275e-01	5.7854e-07
1.8	1.1994e+00	1.4139e-05
2.0	1.6613e+00	2.3129e-05
2.2	2.2135e+00	3.1700e-05

2.4	2.8765e+00	4.3263e-05
2.6	3.6784e+00	6.0933e-05
2.8	4.6586e+00	9.0995e-05
3.0	5.8740e+00	1.4816e-04

§**5.6 Problem 6a.** Apply the Predictor-Corrector algorithm with the correction iterated 2 and then 3 times. Which is better?

This requires that you modify the precor54.m file. For two iterations, the lines

```
k=feval(Fun,pc4T(i)+h,w);
w=pc4W(i)+(h/24)*(9*k+19*k_i-5*k_i1+k_i2);
```

should be repeated. That is, the file pc54_2.m should have

```
k=feval(Fun,pc4T(i)+h,w);
w=pc4W(i)+(h/24)*(9*k+19*k_i-5*k_i1+k_i2);
k=feval(Fun,pc4T(i)+h,w);
w=pc4W(i)+(h/24)*(9*k+19*k_i-5*k_i1+k_i2);
```

. For three iterations, have the two lines repeated once more to make the mfile pc54_3.m

```
k=feval(Fun,pc4T(i)+h,w);
w=pc4W(i)+(h/24)*(9*k+19*k_i-5*k_i1+k_i2);
k=feval(Fun,pc4T(i)+h,w);
w=pc4W(i)+(h/24)*(9*k+19*k_i-5*k_i1+k_i2);
k=feval(Fun,pc4T(i)+h,w);
w=pc4W(i)+(h/24)*(9*k+19*k_i-5*k_i1+k_i2);
```

The first of the repeated lines takes the predicted value of w_{i+1} and evaluates $f(t_{i+1}, w_{i+1})$. The second line then uses that computation in the Adams-Moulton Three-Step Method to give the corrected value. In the first case above, this corrected value becomes a new predicted valued that is again corrected by the Adams-Moulton Three-Step Method. This is repeated once more for the second case.

```
t0=1;tlast=2;w0=1;Fun='fnv4p4a';N=10;
[T,W2]=pc54_2(Fun,t0,tlast,w0,N);
[T,W3]=pc54_3(Fun,t0,tlast,w0,N);
t=1:.01:2;y=solv4p4a(t);Y=solv4p4a(T);
plot(T,W2,'o',T,W3,'+',t,y);[T,W2,abs(W2-Y),W3,abs(W3-Y)]
```

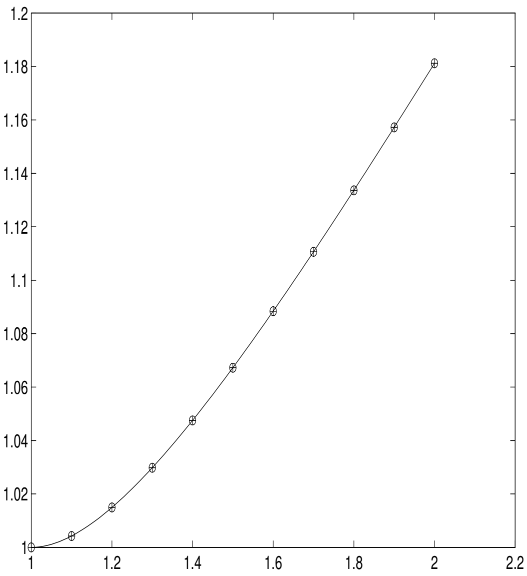
T	W2	abs(W2-Y)	W3	abs(W3-Y)
1.0	1.0000e+00	0	1.0000e+00	0
1.1	1.0043e+00	2.2415e-07	1.0043e+00	2.2415e-07
1.2	1.0150e+00	3.1076e-07	1.0150e+00	3.1076e-07
1.3	1.0298e+00	3.4637e-07	1.0298e+00	3.4637e-07
1.4	1.0475e+00	8.4639e-06	1.0475e+00	8.5002e-06
1.5	1.0672e+00	1.2436e-05	1.0672e+00	1.2482e-05
1.6	1.0884e+00	1.4526e-05	1.0884e+00	1.4575e-05
1.7	1.1106e+00	1.5652e-05	1.1106e+00	1.5701e-05
1.8	1.1336e+00	1.6273e-05	1.1336e+00	1.6322e-05
1.9	1.1572e+00	1.6619e-05	1.1572e+00	1.6668e-05
2.0	1.1812e+00	1.6815e-05	1.1812e+00	1.6864e-05

The error from two iterations is actually smaller than from three iterations!

§**5.6 Problem 6b.** Apply the Predictor-Corrector algorithm with the correction iterated 2 and then 3 times. Which is better? (See the previous problem for set-up notes.)

```
t0=1;tlast=3;w0=0;Fun='fnv4p4b';N=10;
[T,W2]=pc54_2(Fun,t0,tlast,w0,N);
[T,W3]=pc54_3(Fun,t0,tlast,w0,N);
t=1:.01:3;y=solv4p4b(t);Y=solv4p4b(T);
plot(T,W2,'o',T,W3,'+',t,y);[T,W2,abs(W2-Y),W3,abs(W3-Y)]
```

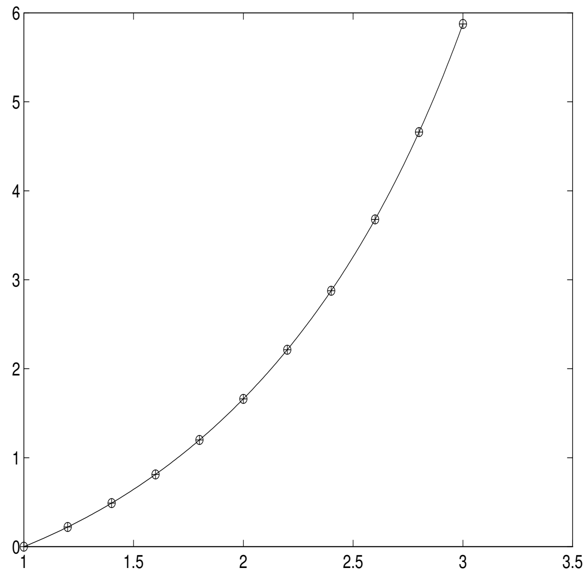
T	W2	abs(W2-Y)	W3	abs(W3-Y)
1.0	0	0	0	0
1.2	2.2125e-01	2.9345e-06	2.2125e-01	2.9345e-06
1.4	4.8968e-01	2.5027e-06	4.8968e-01	2.5027e-06
1.6	8.1275e-01	5.7854e-07	8.1275e-01	5.7854e-07
1.8	1.1995e+00	2.3014e-05	1.1995e+00	2.6625e-05
2.0	1.6613e+00	6.0964e-05	1.6614e+00	6.9133e-05



Plot for §5.6 Problem 6a.

2.2	2.2136e+00	1.2424e-04	2.2136e+00	1.3963e-04
2.4	2.8768e+00	2.3213e-04	2.8768e+00	2.5975e-04
2.6	3.6789e+00	4.1845e-04	3.6789e+00	4.6755e-04
2.8	4.6594e+00	7.4667e-04	4.6595e+00	8.3479e-04
3.0	5.8754e+00	1.3422e-03	5.8756e+00	1.5045e-03

Again two iterations are better than three iterations.



Plot for §5.6 Problem 6b.

§**5.7 Problem 4a.** Here the Variable Step Size Algorithm is applied to the problems cited in the first solution of this Assignment. Here $TOL = 10e-6$, $hmax = 0.5$, and $hmin = 0.02$. (With pc55.m, I got hmin violated. Also when I used $TOL = .000001$ rather than 10e-6 which is really strange! In such cases you could lower $hmin$ and try again.)

```
t0=1;tlast=4;w0=1;Fun='fnv4p4a';TOL=10e-6;hmax=0.5;hmin=0.02;
[T,W]=precor55(Fun, t0, tlast, w0, hmax, hmin, TOL);
t=1:.01:4;y=solv4p4a(t);Y=solv4p4a(T);
plot(T,W,'o',t,y);[T,W,abs(W-Y)]
```

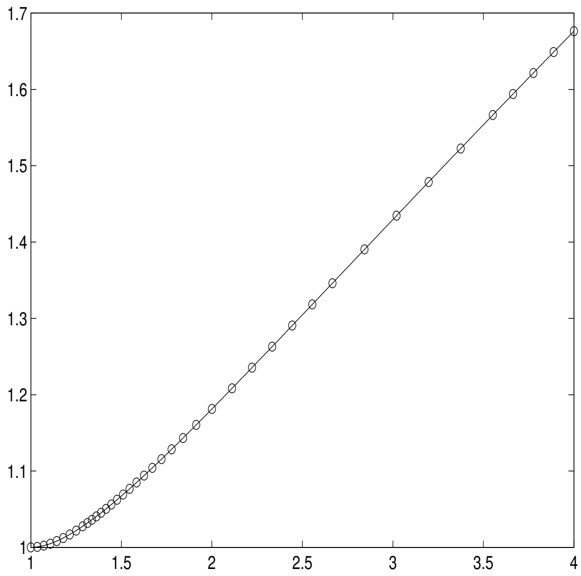
T	W	abs(W-Y)
1.0000e+00	1.0000e+00	0

1.0358e+00	1.0006e+00	1.5597e-09
1.0716e+00	1.0023e+00	2.6615e-09
1.1074e+00	1.0049e+00	3.4504e-09
1.1431e+00	1.0083e+00	1.9714e-07
1.1789e+00	1.0123e+00	3.3514e-07
1.2147e+00	1.0169e+00	4.3534e-07
1.2505e+00	1.0220e+00	5.0861e-07
1.2863e+00	1.0276e+00	5.6255e-07
1.3130e+00	1.0320e+00	5.5587e-07
1.3375e+00	1.0362e+00	5.5018e-07
1.3623e+00	1.0406e+00	5.4478e-07
1.3883e+00	1.0453e+00	5.3952e-07
1.4157e+00	1.0505e+00	5.3434e-07
1.4450e+00	1.0562e+00	5.2924e-07
1.4761e+00	1.0624e+00	5.2422e-07
1.5094e+00	1.0692e+00	5.1929e-07
1.5452e+00	1.0767e+00	5.1445e-07
1.5839e+00	1.0849e+00	5.0970e-07
1.6258e+00	1.0941e+00	5.0507e-07
1.6715e+00	1.1042e+00	5.0055e-07
1.7218e+00	1.1156e+00	4.9616e-07
1.7778e+00	1.1285e+00	4.9194e-07
1.8410e+00	1.1432e+00	4.8793e-07
1.9137e+00	1.1605e+00	4.8423e-07
2.0003e+00	1.1813e+00	4.8108e-07
2.1113e+00	1.2083e+00	4.7952e-07
2.2222e+00	1.2356e+00	4.7827e-07
2.3331e+00	1.2631e+00	4.7738e-07
2.4441e+00	1.2907e+00	6.0966e-07
2.5550e+00	1.3183e+00	7.0235e-07
2.6659e+00	1.3461e+00	7.6892e-07
2.8431e+00	1.3903e+00	3.0027e-06
3.0203e+00	1.4346e+00	1.2976e-06
3.1975e+00	1.4787e+00	1.5407e-06
3.3747e+00	1.5227e+00	1.6852e-06
3.5519e+00	1.5664e+00	1.7861e-06
3.6639e+00	1.5940e+00	1.7931e-06
3.7759e+00	1.6215e+00	1.8005e-06
3.8880e+00	1.6489e+00	1.8082e-06
4.0000e+00	1.6762e+00	1.8194e-06

§**5.7 Problem 4b.** Here the Variable Step Size Algorithm is applied to the problems cited in the first solution of this Assignment. Here $TOL = 10e-6$, $hmax = 0.5$, and $hmin = 0.02$.

```
t0=1;tlast=3;w0=0;Fun='fnv4p4b';TOL=10e-6;hmax=0.5;hmin=0.02;
[T,W]=precor55(Fun, t0, tlast, w0, hmax, hmin, TOL);
t=1:.01:3;y=solv4p4b(t);Y=solv4p4b(T);
plot(T,W,'o',t,y);[T,W,abs(W-Y)]
```

T	W	abs(W-Y)
1.0000e+00	0	0
1.0793e+00	8.2569e-02	4.8920e-08
1.1587e+00	1.7190e-01	7.8761e-08
1.2380e+00	2.6843e-01	9.3478e-08
1.3174e+00	3.7259e-01	2.6425e-07
1.3967e+00	4.8484e-01	4.2411e-07
1.4760e+00	6.0564e-01	5.8549e-07
1.5554e+00	7.3552e-01	7.6143e-07
1.6347e+00	8.7505e-01	9.6110e-07
1.7141e+00	1.0248e+00	1.1931e-06



§5.7 Problem 4a.

1.7934e+00	1.1856e+00	1.4662e-06
1.8727e+00	1.3580e+00	1.7903e-06
1.9521e+00	1.5431e+00	2.1768e-06
2.0314e+00	1.7416e+00	2.6393e-06
2.1108e+00	1.9548e+00	3.1943e-06
2.1768e+00	2.1442e+00	3.4482e-06
2.2429e+00	2.3454e+00	3.7245e-06
2.3089e+00	2.5592e+00	4.0257e-06
2.3749e+00	2.7865e+00	4.6275e-06
2.4410e+00	3.0284e+00	5.3332e-06
2.4955e+00	3.2400e+00	5.7376e-06
2.5500e+00	3.4631e+00	6.1773e-06
2.6045e+00	3.6985e+00	6.6560e-06
2.6590e+00	3.9471e+00	7.4040e-06
2.7135e+00	4.2099e+00	8.2557e-06
2.7591e+00	4.4415e+00	8.8291e-06
2.8048e+00	4.6846e+00	9.4495e-06
2.8504e+00	4.9398e+00	1.0122e-05
2.8960e+00	5.2082e+00	1.1042e-05
2.9416e+00	5.4905e+00	1.2070e-05
2.9872e+00	5.7878e+00	1.3216e-05
2.9904e+00	5.8093e+00	1.3285e-05
2.9936e+00	5.8308e+00	1.3354e-05
2.9968e+00	5.8524e+00	1.3425e-05
3.0000e+00	5.8741e+00	1.3495e-05

§5.7 Problem 5a. The initial value problem is

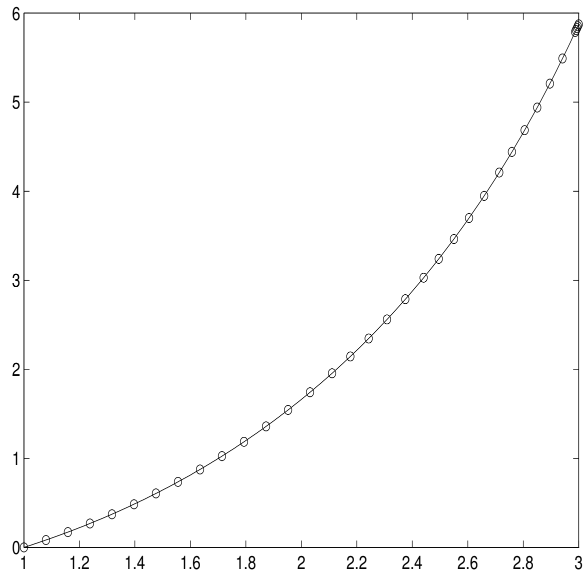
$$y' = -\frac{1}{t}(\sin(t) + y) + \cos(t^2), \quad \sqrt{\pi} \leq t \leq \pi, \quad y(\sqrt{\pi}) = 0.$$

Using $TOL = 10e-5$, $hmax = 0.2$ and $hmin = 0.02$, solve the problem with the variable step size predictor-corrector. We need a function file

```
function z=fnv7p5a(t,y);
% The function f(t,y)=(1/t)(sin(t)+y)+cos(t^2)
z=-(1./t).*(sin(t)+y)+ cos(t.^2);
```

and the file corresponding to the actual solution

```
function z=solv7p5a(t,y);
z=(1./t).*(cos(t)-cos(sqrt(pi))+0.5*sin(t.^2));
```

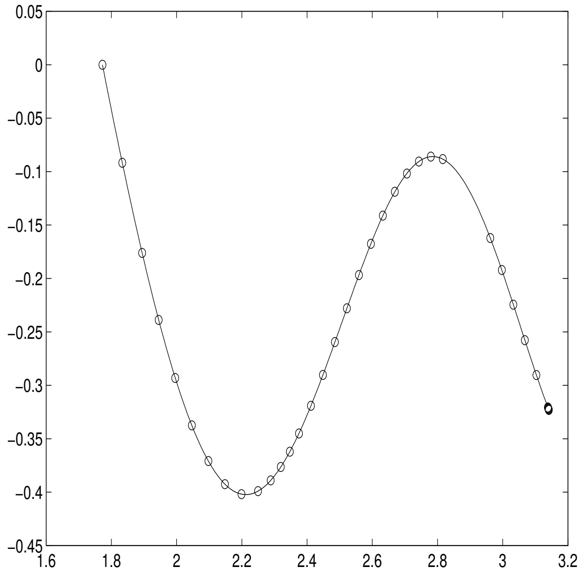


Plot for §5.7 Problem 4b.

```
t0=sqrt(pi);tlast=pi;w0=0;Fun='fnv7p5a';TOL=10e-5;hmax=0.2;hmin=0.02;
[T,W]=precor55(Fun, t0, tlast, w0, hmax, hmin, TOL);
t=t0:.005:tlast;y=solv7p5a(t);Y=solv7p5a(T);
plot(T,W,'o',t,y);[T,W,abs(Y-W)]
```

T	W	abs(Y-W)
1.7725e+00	0	1.5982e-16
1.8334e+00	-9.1744e-02	3.9308e-08
1.8943e+00	-1.7609e-01	1.0019e-07
1.9450e+00	-2.3882e-01	1.3081e-07
1.9958e+00	-2.9307e-01	1.6722e-07
2.0466e+00	-3.3748e-01	2.0776e-07
2.0974e+00	-3.7090e-01	3.0357e-06
2.1481e+00	-3.9248e-01	5.6573e-06
2.1989e+00	-4.0182e-01	7.8783e-06
2.2497e+00	-3.9898e-01	9.4703e-06
2.2884e+00	-3.8897e-01	9.3165e-06
2.3194e+00	-3.7642e-01	9.1932e-06
2.3472e+00	-3.6210e-01	9.0849e-06
2.3751e+00	-3.4502e-01	8.9777e-06
2.4119e+00	-3.1914e-01	8.8373e-06
2.4487e+00	-2.9026e-01	8.6979e-06
2.4854e+00	-2.5947e-01	8.5593e-06
2.5222e+00	-2.2792e-01	7.4307e-06
2.5590e+00	-1.9686e-01	6.1537e-06
2.5957e+00	-1.6753e-01	4.7590e-06
2.6325e+00	-1.4116e-01	3.2907e-06
2.6692e+00	-1.1889e-01	1.8031e-06
2.7060e+00	-1.0174e-01	3.5974e-07
2.7428e+00	-9.0564e-02	9.6892e-07
2.7795e+00	-8.5975e-02	2.1079e-06
2.8163e+00	-8.8328e-02	2.9816e-06
2.9619e+00	-1.6216e-01	3.2903e-06
2.9973e+00	-1.9211e-01	3.2390e-06
3.0326e+00	-2.2447e-01	3.1831e-06
3.0680e+00	-2.5775e-01	1.3893e-06
3.1033e+00	-2.9036e-01	6.7389e-07
3.1386e+00	-3.2066e-01	2.9260e-06

3.1394e+00	-3.2126e-01	2.9253e-06
3.1401e+00	-3.2186e-01	2.9246e-06
3.1409e+00	-3.2245e-01	2.9239e-06
3.1416e+00	-3.2304e-01	2.9232e-06



Plot for §5.7 Problem 5a.