Assignment #2 Solutions

§5.4 Problem 4a. The following inital value problem is used in problems 4a, 6a, 8a, and 10a:

$$y' = \frac{y}{t} - \left(\frac{y}{t}\right)^2, \qquad 1 \le t \le 2, \quad y(1) = 1.$$

You would need to write a program to call this function. In MATLAB, such a program could be called fnv4p4a.m and might look like:

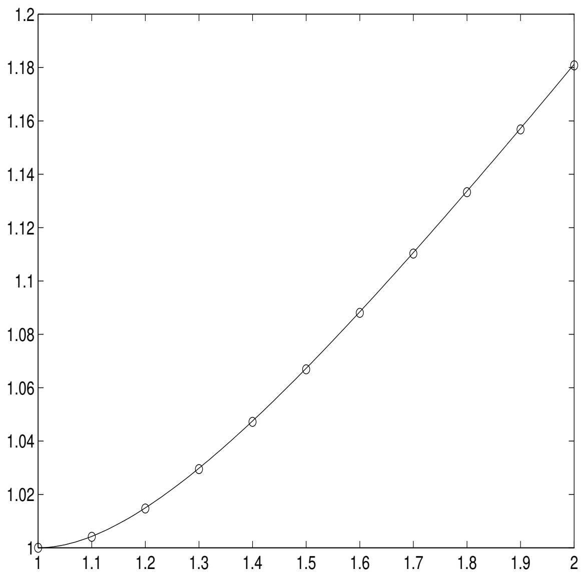
```
function z=fnv4p4a(t,y);
% The function f(t,y)=y/t -(y/t)^ 2
z=y./t-(y./t).^ 2;
```

You will also need a program to call the actual solution for comparison purposes. Again such a MATLAB program, called solv4p4a.m, could be

```
function z=solv4p4a(t);
% The actual solution to y'= f(t,y)=y/t -(y/t)  2
z=t./(1+log(t));
```

Apply the Modified Euler Method to approximate the solution with $h = 0.1$ and compare the results to the actual value. Since $h = 0.1$ and $[a, b] = [1, 2]$, we find $N = (2-1)/.1 = 10$. We use the program modeul_2.m (assuming the files above):

```
t0=1;tlast=2;w0=1;Fun='fnv4p4a';N=10;
[T,W]=modeul_2(Fun,t0,tlast,w0,N);
t=1:.01:2;y=solv4p4a(t);Y=solv4p4a(T);
plot(T,W,'o',t,y);[T,W,abs(W-Y)]
```



T	W	abs(W-Y)
1.0000e+00	1.0000e+00	0
1.1000e+00	1.0041e+00	1.4950e-04
1.2000e+00	1.0147e+00	2.3864e-04
1.3000e+00	1.0295e+00	2.9400e-04
1.4000e+00	1.0472e+00	3.2955e-04
1.5000e+00	1.0669e+00	3.5304e-04
1.6000e+00	1.0881e+00	3.6895e-04
1.7000e+00	1.1103e+00	3.7999e-04
1.8000e+00	1.1333e+00	3.8782e-04
1.9000e+00	1.1568e+00	3.9350e-04
2.0000e+00	1.1808e+00	3.9775e-04

§**5.4 Problem 4b.** The initial value problem is

$$y' = 1 + \frac{y}{t} + \left(\frac{y}{t}\right)^2, \qquad 1 \le t \le 3, \quad y(1) = 0.$$

Hence, you will need to call a program, say fnv4p4b.m in MATLAB. Something like:

```
function z=fnv4p4b(t,y);
% The function f(t,y)=1+y/t +(y/t)^ 2
z=1+y./t+(y./t).^ 2;
```

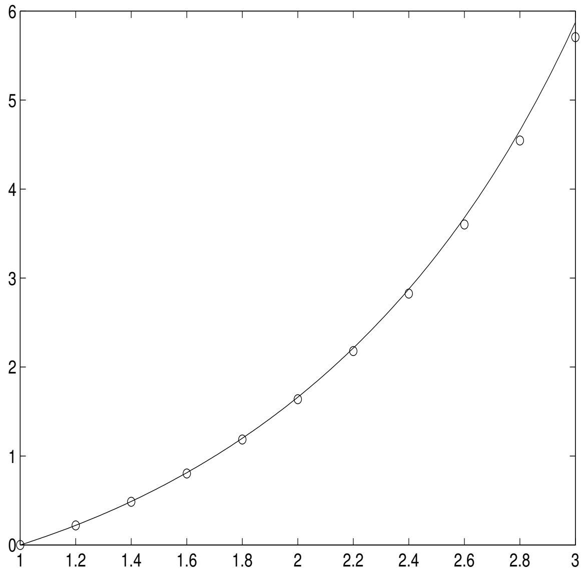
You will also need a program to call the actual solution for comparison purposes; e.g. a MATLAB program called solv4p4b.m like

```
function z=solv4p4b(t);
z=t.*tan(log(t));
```

Apply the Modified Euler Method to approximate the solution with $h = 0.2$ and compare the results to the actual value. Since $h = 0.1$ and $[a, b] = [1, 3]$, we find $N = (3-1)/.2 = 10$. We use the program modeul_2.m (assuming the files above):

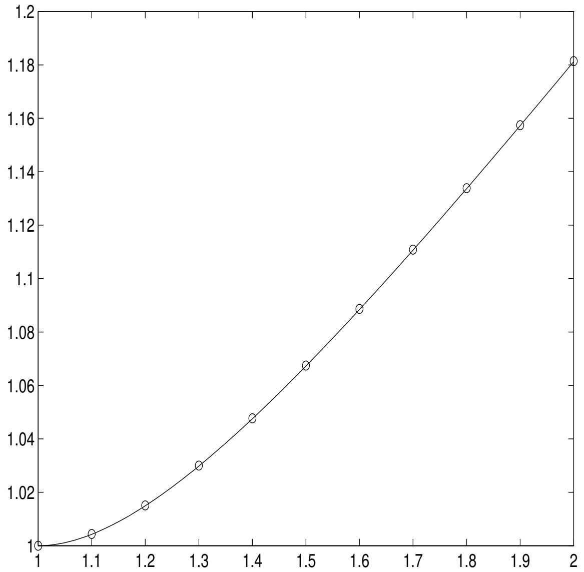
```
t0=1;tlast=3;w0=0;Fun='fnv4p4b';N=10;
[T,W]=modeul_2(Fun,t0,tlast,w0,N);
t=1:.01:3;y=solv4p4b(t);Y=solv4p4b(T);
plot(T,W,'o',t,y);[T,W,abs(W-Y)]
```

T	W	abs(W-Y)
1.0000e+00	0	0
1.2000e+00	2.1944e-01	1.7983e-03
1.4000e+00	4.8505e-01	4.6322e-03
1.6000e+00	8.0401e-01	8.7411e-03
1.8000e+00	1.1849e+00	1.4583e-02
2.0000e+00	1.6384e+00	2.2859e-02
2.2000e+00	2.1789e+00	3.4625e-02
2.4000e+00	2.8251e+00	5.1486e-02
2.6000e+00	3.6025e+00	7.5951e-02
2.8000e+00	4.5466e+00	1.1205e-01
3.0000e+00	5.7076e+00	1.6653e-01



§**5.4 Problem 6a.** Use Heun's Method on the problem in 4a.

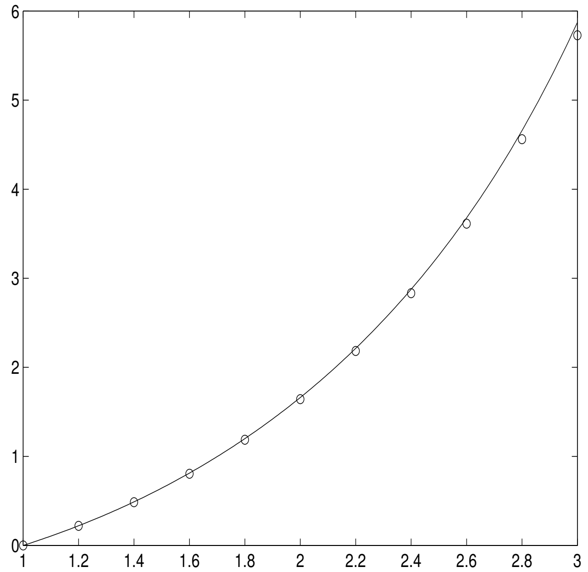
```
t0=1;tlast=2;w0=1;Fun='fnv4p4a';N=10;
[T,W]=heun_2(Fun,t0,tlast,w0,N);
t=1:.01:2;y=solv4p4a(t);Y=solv4p4a(T);
plot(T,W,'o',t,y);[T,W,abs(W-Y)]
```



T	W	abs(W-Y)
1.0000e+00	1.0000e+00	0
1.1000e+00	1.0044e+00	1.1280e-04
1.2000e+00	1.0151e+00	1.5999e-04
1.3000e+00	1.0300e+00	1.7997e-04
1.4000e+00	1.0477e+00	1.8794e-04
1.5000e+00	1.0675e+00	1.9044e-04
1.6000e+00	1.0886e+00	1.9044e-04
1.7000e+00	1.1108e+00	1.8937e-04
1.8000e+00	1.1338e+00	1.8791e-04
1.9000e+00	1.1574e+00	1.8641e-04
2.0000e+00	1.1814e+00	1.8502e-04

§**5.4 Problem 6b.** Use Heun's Method on the problem in 4b.

```
t0=1;tlast=3;w0=0;Fun='fnv4p4b';N=10;
[T,W]=heun_2(Fun,t0,tlast,w0,N);
t=1:.01:3;y=solv4p4b(t);Y=solv4p4b(T);
plot(T,W,'o',t,y);[T,W,abs(W-Y)]
```

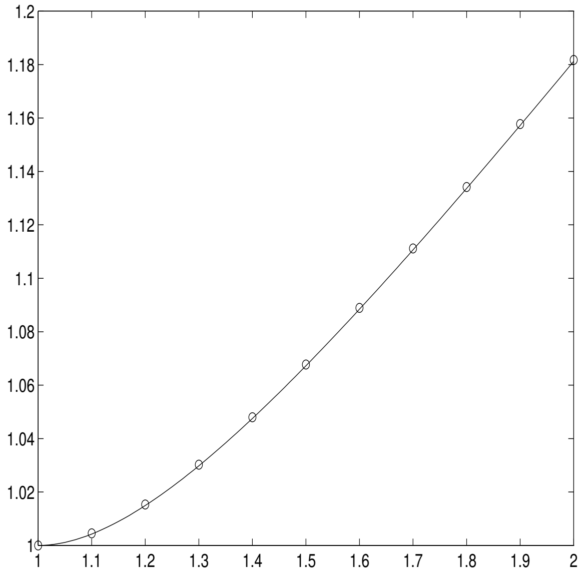


T	W	abs(W-Y)
1.0000e+00	0	0
1.2000e+00	2.1972e-01	1.5196e-03
1.4000e+00	4.8583e-01	3.8502e-03
1.6000e+00	8.0550e-01	7.2509e-03
1.8000e+00	1.1873e+00	1.2139e-02
2.0000e+00	1.6421e+00	1.9143e-02

2.2000e+00	2.1843e+00	2.9207e-02
2.4000e+00	2.8328e+00	4.3779e-02
2.6000e+00	3.6133e+00	6.5133e-02
2.8000e+00	4.5617e+00	9.6959e-02
3.0000e+00	5.7286e+00	1.4548e-01

§**5.4 Problem 8a.** Midpoint Method on the problem in 4a.

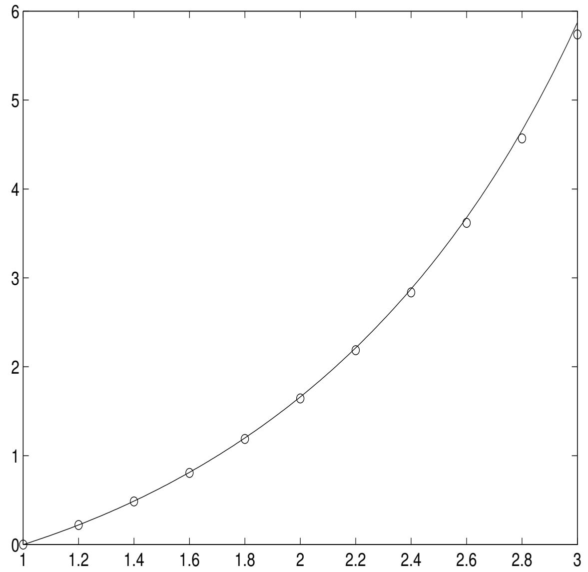
```
t0=1;tlast=2;w0=1;Fun='fnv4p4a';N=10;
[T,W]=rkmid_2(Fun,t0,tlast,w0,N);
t=1:.01:2;y=solv4p4a(t);Y=solv4p4a(T);
plot(T,W,'o',t,y);[T,W,abs(W-Y)]
```



T	W	abs(W-Y)
1.0000e+00	1.0000e+00	0
1.1000e+00	1.0045e+00	2.5342e-04
1.2000e+00	1.0153e+00	3.7335e-04
1.3000e+00	1.0302e+00	4.3332e-04
1.4000e+00	1.0480e+00	4.6426e-04
1.5000e+00	1.0677e+00	4.8039e-04
1.6000e+00	1.0889e+00	4.8868e-04
1.7000e+00	1.1111e+00	4.9276e-04
1.8000e+00	1.1341e+00	4.9457e-04
1.9000e+00	1.1577e+00	4.9519e-04
2.0000e+00	1.1817e+00	4.9524e-04

§**5.4 Problem 8b.** Midpoint Method on the problem in 4b.

```
t0=1;tlast=3;w0=0;Fun='fnv4p4b';N=10;
[T,W]=rkmid_2(Fun,t0,tlast,w0,N);
t=1:.01:3;y=solv4p4b(t);Y=solv4p4b(T);
plot(T,W,'o',t,y);[T,W,abs(W-Y)]
```

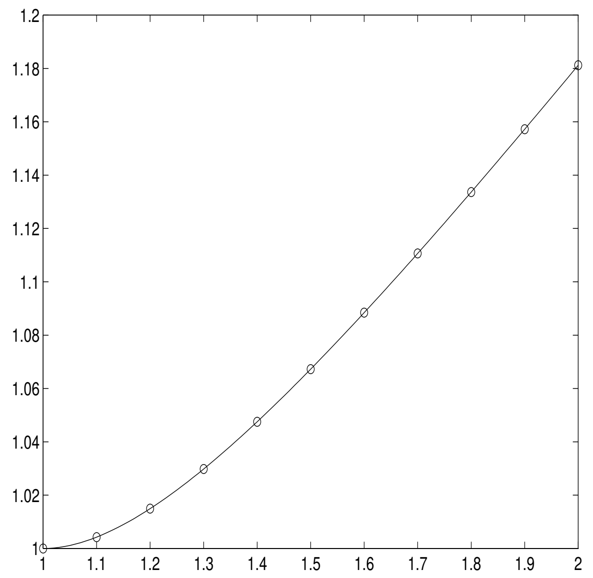


T	W	abs(W-Y)
1.0000e+00	0	0
1.2000e+00	2.1983e-01	1.4081e-03
1.4000e+00	4.8618e-01	3.5046e-03
1.6000e+00	8.0618e-01	6.5678e-03
1.8000e+00	1.1884e+00	1.0999e-02
2.0000e+00	1.6439e+00	1.7393e-02
2.2000e+00	2.1869e+00	2.6641e-02
2.4000e+00	2.8364e+00	4.0116e-02
2.6000e+00	3.6185e+00	5.9983e-02
2.8000e+00	4.5689e+00	8.9771e-02
3.0000e+00	5.7386e+00	1.3545e-01

§**5.4 Problem 10a.** Runge-Kutta Order 4 on the problem 4a.

```
t0=1;tlast=2;w0=1;Fun='fnv4p4a';N=10;
[T,W]=rk4(Fun,t0,tlast,w0,N);
t=1:.01:2;y=solv4p4a(t);Y=solv4p4a(T);
plot(T,W,'o',t,y);[T,W,abs(W-Y)]
```

T	W	abs(W-Y)
1.0	1.00000000000000	0

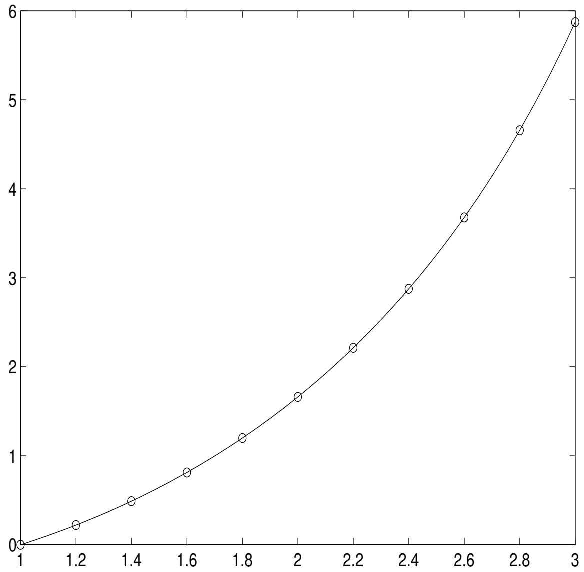


1.1	1.00428150378955	0.00000022414665
1.2	1.01495200327427	0.00000031075947
1.3	1.02981334258920	0.00000034636878
1.4	1.04753355826668	0.00000036098584
1.5	1.06726198780005	0.00000036638182
1.6	1.08843231933485	0.00000036761094
1.7	1.11065468515655	0.00000036698972
1.8	1.13365319110587	0.00000036562744
1.9	1.15722806898698	0.00000036406769
2.0	1.18123185572281	0.00000036257647

§**5.4 Problem 10b.** Runge-Kutta Order 4 on the problem 4b.

```
t0=1;tlast=3;w0=0;Fun='fnv4p4b';N=10;
[T,W]=rk4(Fun,t0,tlast,w0,N);
t=1:.01:3;y=solv4p4b(t);Y=solv4p4b(T);
plot(T,W,'o',t,y);[T,W,abs(W-Y)]
```

T	W	abs(W-Y)
1.0	0	0
1.2	0.22124570728881	0.00000293453118
1.4	0.48968416644337	0.00000250269242
1.6	0.81275216202401	0.00000057853753
1.8	1.19943202163924	0.00000661868670
2.0	1.66126511544869	0.00001664027288
2.2	2.21346931657636	0.00003249690427
2.4	2.87649411454154	0.00005730545330
2.6	3.67837899609925	0.00009633475260

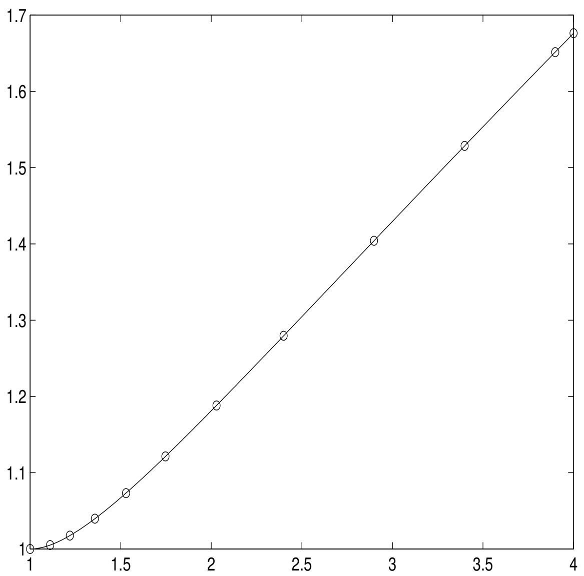


2.8	4.65850628368228	0.00015877455723
3.0	5.87383856983279	0.00026140835138

§**5.5 Problem 4a.** Runge-Kutta-Fehlberg on the function in Problem §5.4p4a, but on the interval $1 \leq t \leq 4$.

```
t0=1;tlast=4;w0=1;Fun='fnv4p4a';TOL=.000001;hmax=0.5;hmin=0.05;
[T,W]=rkf53_2(Fun, t0, tlast, w0, hmax, hmin, TOL);
t=1:.01:4;y=solv4p4a(t);Y=solv4p4a(T);
plot(T,W,'o',t,y);[T,W,abs(W-Y)]
```

T	W	abs(W-Y)
1.00000000000000	1.00000000000000	0
1.11019457034243	1.00512372683806	0.00000005008344
1.21913138202213	1.01752120742417	0.00000006387802
1.35726943760421	1.03967493157278	0.00000008068670
1.52901118020475	1.07327567199058	0.00000009647435
1.74705835949435	1.12139484998659	0.00000011067125
2.02864159746852	1.18817015373962	0.00000012109483
2.39943496134323	1.27953957688418	0.00000012387333
2.89851474482351	1.40418426236546	0.00000011211799
3.39851474482351	1.52856391344281	0.00000011162287
3.89851474482351	1.65149631598792	0.00000011352300
4.00000000000000	1.67623925076970	0.00000011398408

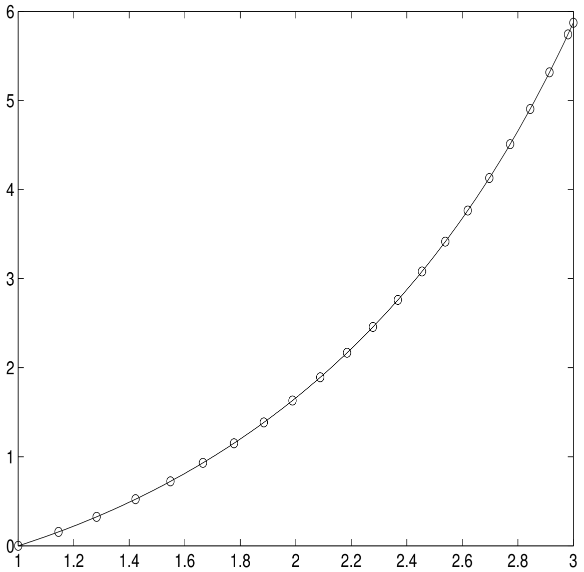


§5.5 Problem 4b. Runge-Kutta-Fehlberg on the function in Problem §5.4p4b.

```
t0=1;tlast=3;w0=0;Fun='fnv4p4b';TOL=.000001;hmax=0.5;hmin=0.05;
[T,W]=rkf53_2(Fun, t0, tlast, w0, hmax, hmin, TOL);
t=1:.01:3;y=solv4p4b(t);Y=solv4p4b(T);
plot(T,W,'o',t,y);[T,W,abs(W-Y)]
```

T	W	abs(W-Y)
1.00000000000000	0	0
1.14502651480180	0.15602352312343	0.00000008102342
1.28223031859482	0.32549720741680	0.00000017367989
1.42257522379408	0.52326437309877	0.00000031544923
1.54822381143641	0.72341231930471	0.00000044575436
1.66557587977610	0.93202879220458	0.00000058195282
1.77736543141438	1.15214799745949	0.00000073106898
1.88472255233318	1.38512344891532	0.00000089669908
1.98819979091608	1.63169513988363	0.00000108149376
2.08809973108002	1.89233027458687	0.00000128781571
2.18460242436023	2.16735137174972	0.00000151796040
2.27782444680326	2.45699493970325	0.00000177424846
2.36784933193568	2.76144230263507	0.00000205907100
2.45474424461636	3.08083740445332	0.00000237491450
2.53856940067251	3.41529782699488	0.00000272437569
2.61938340888053	3.76492197266227	0.00000311017111
2.69724621734110	4.12979393550133	0.00000353514355
2.77222061203949	4.50998690523663	0.00000400226662
2.84437282816275	4.90556560040360	0.00000451464803

2.91377261842170	5.31658803721503	0.00000507553201
2.98049299565851	5.74310683141331	0.00000568830113
3.00000000000000	5.87410585076672	0.00000587258255



§**5.5 Problem 5b.** Runge-Kutta-Fehlberg on the problem

$$y' = 1 + t(\sin(ty)), \quad 0 \le t \le 2, \quad y(0) = 0,$$

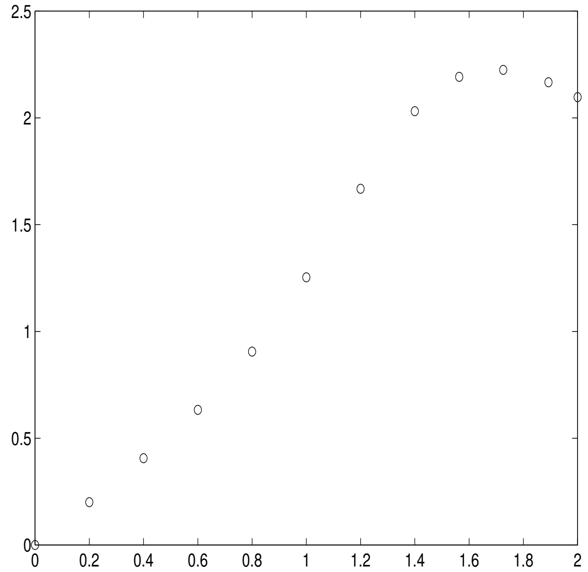
using $TOL = .0001$, and $hmax = 0.2$. We need a function file

```
function z=fnv5p5b(t,y);
z=1+t.*(sin(t.*y));
```

Then we apply the algorithm

```
t0=0;tlast=2;w0=0;Fun='fnv5p5b';TOL=.0001;hmax=0.2;hmin=0.05;
[T,W]=rkf53_2(Fun, t0, tlast, w0, hmax, hmin, TOL);
plot(T,W,'o');[T,W]
```

T	W
0	0
0.20000000000000	0.20040044880470
0.40000000000000	0.40644485344092
0.60000000000000	0.63302295376085
0.80000000000000	0.90555637531214
1.00000000000000	1.25337967327815
1.20000000000000	1.66797187962042
1.40000000000000	2.03142803430059
1.56375335262661	2.19211296637011
1.72593890588638	2.22501250651918



1.89247856430546	2.16707831788043
2.00000000000000	2.09731552103111