PIMS–AMI Workshop on Applied Harmonic Analysis

August 15, 2017

University of Alberta, Canada

Conference Venue

Venue: SAB 331 (South Academic Building), North Campus, University of Alberta

Organizers

Bin Han, University of Alberta

Rongqing Jia, University of Alberta

Yaushu Wong, University of Alberta

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Conference Schedule

August 15, Tuesday

9:00-11:40 Chair: Rongqing Jia

9:00-9:50 **Fangliang He**,
Quantifying Spatial Distribution of Species

9:50-10:40 **Zhiqiang Xu**,
Low-rank Matrix Recovery and Phase Retrieval

10:50-11:40 **Feng Dai**,
Chebyshev-type Cubature Formulas for Doubling Weights on Spheres, Balls and Simplexes

14:00-16:50 Chair: Yaushu Wong

14:00-14:50 **Xiaosheng Zhuang**,
Multiscale Data Analysis: Framelets, Manifolds and Graphs

14:50-15:40 **Yi Shen**,
Some Frame-based Noncovex Methods for Image Restoration

15:50-16:20 **Wenrui Ye**,
Bochner-Riesz Means for the Dunkl Transforms

16:20-16:50 **Chenzhe Diao**,
On Quasi-tight Framelets

Abstracts

Chebyshev-type Cubature Formulas for Doubling Weights on Spheres, Balls and Simplexes

Feng Dai

University of Alberta

In this talk, I will report my recent joint work with Han Feng on strict Chebyshev-type cubature formulas (CF) (i.e., equal weighted CFs) for doubling weights w on the unit sphere $\mathbb{S}^{d-1}$ of $\mathbb{R}^d$ equipped with the usual surface Lebesgue measure $d\sigma_d$ and geodesic distance $\mathrm{dist}(\cdot,\cdot)$. Our main interest is on the minimal number $\mathcal{N}_n(wd\sigma_d)$ of nodes required in a strict Chebyshev-type CF of degree n for a doubling measure $wd\sigma_d$ on $\mathbb{S}^{d-1}$. One of our main results states that for a doubling weight w on $\mathbb{S}^{d-1}$,

$$\mathcal{N}_n(wd\sigma_d) \sim \mu_{n,w} := \max_{x\in\mathbb{S}^{d-1}} \frac{1}{w(B(x,n^{-1}))},$$

where the constants of equivalence are independent of n, and B(x,r) denotes the spherical cap with center $x \in \mathbb{S}^{d-1}$ and radius $r > 0$. It will be shown that given a normalized doubling weight w on $\mathbb{S}^{d-1}$, there exists a positive constant K_w depending only on the doubling constant of w such that for each positive integer n and each integer $N \geq K_w\mu_{n,w}$, there exists a set of N distinct nodes $z_1, \cdots, z_N$ on $\mathbb{S}^{d-1}$ which admits a strict Chebyshev-type cubature formula (CF) of degree n for the measure $w(x)d\sigma_d(x)$, and which satisfies

$$\min_{1\leq i\neq j\leq N} \mathrm{dist}(z_i, z_j) \geq c_* N^{-\frac{1}{d-1}}$$

if in addition $w \in L^\infty(\mathbb{S}^{d-1})$. The proofs of these results rely on new convex partitions of $\mathbb{S}^{d-1}$ that are regular with respect to the weight w. The weighted results on the sphere also allow us to establish similar results on strict Chebyshev-type CFs on the unit ball and the standard simplex of $\mathbb{R}^d$.

Our results generalize the recent results of Bondarenko, Radchenko, and Viazovska on spherical designs.

On Quasi-tight Framelets

Chenzhe Diao

University of Alberta

Wavelet frames, both their theories and applications, have been studied extensively in the literature over the past few decades. The special case of wavelet tight frames, due to their simple structures and many desired properties, are widely used in applications. However, the construction of tight framelet filter banks imposes extra restrictions on the low-pass filter. In order to construct framelet filter banks with arbitary low-pass filters, we generalize the tight framelets to quasi-tight framelets, which allows the frames and their dual to differ by a possibly different sign. I will present some results on the construction of 1d quasi-tight framelet filter banks, as well as the constructon when symmetry is present.

Quantifying Spatial Distribution of Species

Fangliang He

University of Alberta

Biological species in nature distribute in numerous forms, as driven by processes such as dispersal, competition and habitat heterogeneity across a broad range of spatial scales. Measuring the geometry of species distribution is necessary for understanding ecosystem structure and function and for inferring the underlying forces. This is also a critical step for predicting the impact of environmental change and habitat loss on biodiversity. In this talk, I will present empirical cases of species distributions and review the subject (in the field of ecology). I will conclude my talk by proposing some questions that may be of potential interest to mathematicians for possible future collaborations.

Some Frame-based Noncovex Methods for Image Restoration

Yi Shen

Zhejiang Sci-Tech University

Since digital images are usually sparse in the wavelet frame domain, some nonconvex minimization models based on wavelet frame have been proposed and sparse approximations have been widely used in image restoration in recent years. Some proximal alternating iterative hard thresholding methods are proposed in this talk to solve the nonconvex model based on wavelet frame to restore degraded image. We will perform the test on image denoising and image deconvolution.

Low-rank Matrix Recovery and Phase Retrieval

Zhiqiang Xu

Chinese Academy of Sciences

In this talk, we presents several results that address a fundamental question in low-rank matrix recovery: how many measurements are needed to recover low rank matrices? We begin by investigating the complex matrices case and show that $4nr4r^2$ generic measurements are both necessary and sufficient for the recovery of rank-r matrices in $\mathbb{C}^{n\times n}$. Thus, we confirm a conjecture which is raised by Eldar, Needell and Plan for the complex case. We next consider the real case and prove that the bound $4nr4r^2$ is tight provided $n = 2^k + r, k \in \mathbb{Z}_+$. Motivated by Vinzant's work, we construct 11 matrices in $R^{4\times4}$ by computer random search and prove they define injective measurements on rank-1 matrices in $R^{4\times4}$. This disproves the conjecture raised by Eldar, Needell and Plan for the real case.

Bochner-Riesz Means for the Dunkl Transforms

Wenrui Ye

University of International Business and Economics

In this talk, I will mainly describe joint work with Dr. Feng Dai on the critical index for the almost everywhere convergence of the Bochner-Riesz means in weighted L_p-spaces with $p = 1$ or $p > 2$. Our results under the case $p > 2$ are in full analogy with the classical result of M. Christ on estimates of the maximal Bochner-Riesz means of Fourier integrals and the classical result of A. Carbery, Jos L. Rubio De Francia and L. Vega on a.e. convergence of Fourier integrals. Besides, I will also introduce several new results that are related to our main results, including: (i) local restriction theorem for the Dunkl transform which is significantly stronger than the global one, but more difficult to prove; (ii) the weighted Littlewood Paley inequality with Ap-weights in the Dunkl noncommutative setting; (iii) sharp local pointwise estimates of several important kernel functions.

Multiscale Data Analysis: Framelets, Manifolds and Graphs

Xiaosheng Zhuang

City University of Hong Kong

While Big Data are high-volume, high-dimensional, and high complexity, they are typically concentrated on low-dimensional manifolds or can be represented by graphs, digraphs, etc. Sparsity is the key to the successful analysis of data in various forms. Multiscale representation systems provide efficient and sparse representation of various data sets. In this talk, we will discuss the characterizations, construction, and applications of framelets on manifolds and graphs. We shall demonstrate that tight framelets can be constructed on compact Riemannian manifolds or graphs, and fast algorithmic realizations exist for framelet transforms on manifolds and graphs. Explicit construction of tight framelets on the sphere as well as numerical examples will be shown.

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