PIMS / AMI Seminar

Friday, September 26, 2014
3:00 p.m.
CAB 657

"Stochastic Three-Dimensional Rotating Navier-Stokes Equations: Averaging, Convergence and Regularity"

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Abstract

We consider the Stochastic 3D Rotating Navier-Stokes Equations

$$\partial_t U - v\Delta U + (U \cdot \nabla)U + \frac{1}{\epsilon} e_3 \times U = -\nabla\pi + \sqrt{Q}\frac{\partial w}{\partial t}, \quad (1)$$

$$divU = 0, \quad (2)$$

$$U|_{t=0} = U^0(x1, x2, x3), \quad (3)$$

where $U(t,x) = (U_1, U_2, U_3), x = (x1, x2, x3)$ is the velocity field (a random 3D vector field), $\pi(t,x)$ is the pressure (a random scalar field), $v > 0$ is the kinematic viscosity, $\mathcal{W}(t)$ is a cylindrical Wiener process which injects energy in x3 dependent modes, defined on a filtered probability space $(\Omega, F_t, \mathcal{P})$, in the Hilbert space $H = L_s^2$ of square integrable solenoidal vector fields; Q is a non-negative, symmetric, trace class operator in H. In (1), e_3 is the vertical axis, $e_3 \times U = (-U_2, U_1, 0) = JU$; J is the non-local Poincar´e-Coriolis-Riesz rotation operator on solenoidal vector fields. Eqs. (1)-(3) are fundamental in meteorology and geophysical fluid dynamics where the rotation plays an essential role.

For the stochastic three-dimensional rotating Navier-Stokes equations (1)-(3), we prove averaging theorems for stochastic dynamics in the case of strong rotation. Regularity results are established by bootstrapping from global regularity of the limit stochastic equations and convergence theorems. The energy injected in the system by the noise including x3-dependent modes is large, the initial condition has large energy, and the regularization time horizon is long. Regularization is the consequence of precise mechanisms of relevant three-dimensional nonlinear interactions. We establish multiscale averaging and convergence theorems for the stochastic dynamics.

References

[1] Flandoli F. , Mahalov A. , "Stochastic 3D Rotating Navier-Stokes Equations: Averaging, Convergence and Regularity," Archive for Rational Mechanics and Analysis, 205, No. 1, 195–237 (2012).

[2] Cheng B. , Mahalov A. , "Euler Equations on a Fast Rotating Sphere – Time-Averages and Zonal Flows," European Journal of Mechanics B/Fluids, 37, 48-58 (2013).

Refreshments will be served in CAB 649 at 2:30 p.m.