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## QUIZ 6

**Problem 1.** Assume  $\Omega$  is connected. Use maximum principles to show that the only smooth solutions of the Neumann boundary-value problem

$$\begin{cases} -\Delta u = 0 & \text{in } \Omega \\ \frac{\partial u}{\partial \nu} = 0 & \text{on } \partial\Omega \end{cases} \quad (1)$$

are  $u \equiv \text{constant}$ .

**Solution.** Assume  $u$  is not a constant. Then there must be  $x^0$  where  $u$  attains maximum in  $\bar{U}$ . If  $x^0 \in U$ , the strong maximum principle tells us that  $u$  is constant, contradiction. Thus the maximum can only be attained at the boundary, as a consequence  $u(x^0) > u(x)$  for all  $x \in U$ . Now it follows from the Hopf lemma that  $\frac{\partial u}{\partial \nu}(x^0) > 0$ , contradicting the assumption in the problem.