MATH 527 A1 FALL 2009

NAME: _____

QUIZ 2

Problem 1. Let U^+ denote the open half-ball $\{x \in \mathbb{R}^n \mid |x| < 1,\ x_n > 0\}$. Assume $u \in C(\overline{U^+})$ is harmonic in U^+, with $u = 0$ on $\partial U^+ \cap \{x_n = 0\}$. Set

$$v(x) := \begin{cases} u(x) & x_n \geqslant 0 \\ -u(x_1, \ldots, x_{n-1}, -x_n) & x_n < 0 \end{cases} \tag{1}$$

for $x \in U = B^0(0,1)$. Prove v is harmonic in U.

(Hint: First notice that it suffices to show that, for any $x \in U$, there is $r > 0$ such that v is harmonic in $B_r(x)$. Then use the converse of the mean value property)

Solution.