MATH 527 A1 FALL 2009

NAME: _____

QUIZ 1

Problem 1.

a) **(5 pts)** Consider the following transport equation with source:

$$v_t + a(x,t)\, v_x = -\, v^2, \qquad x \in \mathbb{R}, \ t \in [0, \infty); \qquad v(x,0) = v_0(x). \tag{1}$$

where $a(x,t) \in C^1(\mathbb{R} \times [0, \infty))$ and $|v_0(x)| \leqslant M$ for some constant $M < \infty$. Prove that, if $v_0(x) < 0$ for some $x \in \mathbb{R}$, then v cease to be bounded in finite time.

(Hint: Use the method of characteristics.)

b) **(5 pts)** Consider the inviscid Burgers equation

$$u_t + u\, u_x = 0, \qquad x \in \mathbb{R}, \ t \in [0, \infty); \qquad u(x,0) = u_0(x). \tag{2}$$

Assume $u_0(x) \in C^\infty(\mathbb{R})$. Prove that $u(x,t)$ cease to be C^1 in finite time if $u_0'(x) < 0$ for some $x \in \mathbb{R}$.

(Hint: Use part a))

Solution.