

Math 421 Winter 2017 Midterm 2

MAR. 17, 2017 1PM - 1:50PM. TOTAL 30 PTS

NAME:

ID#:

- Please write clearly and **show enough work/explain your reasoning** (this is very important!).
- No electronic devices are allowed.

Question 1. (8 pts) *Let $a_{n+3} = 3 a_{n+2} - 3 a_{n+1} + a_n$ for all $n \geq 0$ and $a_0 = a_1 = 0, a_2 = 1$. Use generating function to find the numerical value of a_{100} .*

Question 2. (7 pts) *Find the number of different ways distributing n different balls to four boxes where an odd number of balls are in the fourth box.*

Question 3. (12 pts) *Find the number of ways to color the eight vertices of a regular octagon with 2 colors, if*

- a) **(6 pts)** *only rotations are allowed;*
- b) **(6 pts)** *both rotation and flipping are allowed.*

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Question 4. (3 pts) *How many ways are there to put 16 identical balls in four identical boxes at the four vertices (one at each vertex) of a square board, allowing empty boxes, assuming that the board can freely rotate? Give your answer in numerical value.*