

MATH 421 Q1 WINTER 2017 HOMEWORK 1 SOLUTIONS

Due Jan. 19, 12pm.

Total 20 points.

QUESTION 1. (5 PTS) *A committee is to be chosen from among 8 scientists, 7 psychics, and 12 clerics. If the committee is to have two members of different backgrounds, how many such committees are there? Justify your answer.*

Solution. We divide the possibilities into three cases and apply the product rule to each.

- No scientist. $7 \times 12 = 84$;
- No psychics. $8 \times 12 = 96$;
- No clerics. $8 \times 7 = 56$.

Thus the total is $84 + 96 + 56 = 236$.

QUESTION 2. (5 PTS) *How many 5-letter words¹ either start with d or do not have the letter d? Justify your answer.*

Solution.

- Start with *d*: For each of the other four positions we have 26 choices. So the total is 26^4 .
- No *d*: For each of the five positions we have 25 choices. So the total is 25^5 .

The answer is then $26^4 + 25^5$.

QUESTION 3. (5 PTS) *In how many ways can we get a sum of 4 when two dice are rolled if the two dice are identical. Justify your answer.*

Solution. As the two dice are identical, the only possibilities are 1, 3 and 2, 2. So there are two ways.

QUESTION 4. (5 PTS) *A composition of a positive integer n is an ordered list of positive integers (called parts) that sum to n . The four compositions of 3 are 3; 2,1; 1,2 and 1,1,1. Obtain a formula for the number of compositions of n . Justify your answers.*

Solution. We consider the following modeling: List the n 1's in a line:

$$1 \quad 1 \quad 1 \quad 1 \cdots 1 \quad 1 \quad 1. \tag{1}$$

There are $n - 1$ empty spots in between the 1's. For each spot we have the choice of putting a + or a , . We see that each sequence of $n - 1$ choices gives a composition. For example, when $n = 3$, the choice $1 + 1 + 1$ gives the composition 3; the choice $1 + 1, 1$ gives the composition 2, 1. Applying the product rule we see that there are total of 2^{n-1} compositions.

1. a "5-letter word" here just means an ordered list of 5 letters. For example abcde is a word.