Math 348 Differential Geometry of Curves and Surfaces

Lecture 1 Introduction

Xinwei Yu

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CAB 527, xinwei2@ualberta.ca
Department of Mathematical & Statistical Sciences
University of Alberta

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Course Information

Course Organization

- **Instructor.**

  Xinwei Yu; CAB 527; xinwei2@ualberta.ca;

- **Course Webpage.**
  - http://www.math.ualberta.ca/ xinweiyu/348.A1.17f
  - We will not use eClass.
- **Getting Help.**
  1. Office Hours. TBA.
  2. Appointments.
  3. Emails.
  4. It is a good idea to form study groups.

Evaluation

- **Homeworks.**
  9 HWs; 20%; Worst mark dropped; No late HW.
  Due dates:
  9/22; 9/29; 10/6; 10/20; 10/27; 11/3; 11/10; 12/1; 12/8.
  12:00 (noon) in assignment box (CAB 3rd floor)
- **Midterms.**
  30%; 10/5; 11/9. One hour. In class.
  No early/make-up midterms.
- **Final Exam.** Tentative.
  50%; 12/14 @9a. Two hours.

$$\geqslant 50\% \Longrightarrow \geqslant D; \geqslant 90\% \Longrightarrow \geqslant A.$$

Resources

- **Textbook.**
  - Andrew Pressley, Elementary Differential Geometry, 2nd Ed., Springer, 2010.
- **Reference books.**
  - Most books calling themselves ”Introductory DG”, ”Elementary DG”, or ”Classical DG”.
- **Online notes.**
  - Lecture notes (updated from Math 348 2016 Notes)
  - Theodore Shifrin lecture notes.
  - Xi Chen lecture notes for Math 348 Fall 2015.
- **Online videos.**
  - Most relevant: Differential Geometry - Claudio Arezzo (ICTP Math).
  - Many more on youtube etc..

Geometry, Differential

- **Geometry.**
  - Originally, ”earth measure”.
  - More precisely, study of shapes.[1]
- **Differential.**
  - Calculus is (somehow) involved.
- **Classical Differential Geometry.**
  - The study of shapes, such as curves and surfaces, using calculus.
  - $\approx$ Math 348.
- **Math 348 Plan.**
  - `Weeks 1--3.` Curves and surfaces in calculus.
  - `Weeks 4--6.` Curves in $\mathbb{R}^2$ and $\mathbb{R}^3$, measurements on surfaces.
  - `Weeks 7--13.` Curvatures of surfaces.

[1] Curves and surfaces–precise meaning later–in Math 348.

Before Classical Differential Geometry

Geometry Before Calculus

- **Prehistory.**
  - Area/volume of simple shapes.
  - Rough estimates of π. [2]
  - Examples of Pythagorean triples.
- **Greek Geometry.**
  - Complete understanding of straight lines and circles (Euclid; Appollonius; Archimedes).
  - Restricted to straight lines and perfect circles.
- **Analytic Geometry.**
  - Geometry powered by (polynomial) algebra.
  - No longer restricted to straight lines and circles.

[2] It may nor may not be understood that π is a single number, e.g., the "perimeter π" may be different from the "area π" or the "volume π".

Geometry as Foundation of Mathematics

- **The Evolution of Foundations of Mathematics.**

  $$\text{Non-exist} \overset{\text{c. 500 B.C.}}{\longrightarrow} \text{Number Theory} \overset{\text{c. 300 B.C.}}{\longrightarrow} \text{Geometry} \overset{\text{c. 1700}}{\longrightarrow} \text{Algebra}$$

- **Geometry as Foundation.**
  - Number: length; Square: Area; Cubic: Volume. [3]
  - Geometric algebra.
    - Euclid: General quadratic equations;
    - Appollonius, Omar Khayyam, etc.: Many cubic equations;
    - Newton: Still tried to argue through geometry
    - Examples: See e.g. "Taming the Unknown: A History of Algebra from Antiquity to the Early Twentieth Century" by Katz and Parshall.

[3] Higher powers: essentially meaningless.

Analytic Geometry: Geometry powered by Algebra

- **Algebra.**
  - "al-jabr": Restoration. Example:

$$3x + 2 = 4 - 2x \Longrightarrow 5x + 2 = 4.$$

  - Great development between 500 A.D. and 1500 A.D..
  - Relation to geometry: Graphs and Equations of polynomials.
- **Analytic Geometry.**
  - "analytic": pretend the unknown is known.[4]
  - "Algebraic geometry" instead of "geometric algebra".

[4] No relation to calculus!

The Success and Limitation of Analytic Geometry

- **Key Idea.**
  Relate properties of geometric shapes to algebraic properties of appropriate algebraic objects.
- **Procedure.**
  1. Represent geometric shapes algebraically.
  2. Study the algebraic equations, i.e., (kind of) solve them.
  3. Interpret the results geometrically.
- `No longer restricted to conic curves.`

Limitation

`Polynomial equations are difficult/impossible to solve.`

Classical DG: Upgrade to Calculus

Calculus

- **What is Calculus.**
  - Calculus: Stones used in Roman abacus.[5]
  - Newton: A new kind of geometry for general curves and surfaces.
  - Lagrange: A generalized algebra for polynomials of infinite degree.
- **Key Ingredients.**
  1. One Object: Function.
  2. Two Operations: Differentiation, Integration.

[5] Wiki: Roman abacus

Classical Differential Geometry

- **Procedure.**
  1. Represent curves and surfaces as functions;
  2. Study these functions through differentiation and integration;
  3. Relate the results to properties of the curves and surfaces.
- **Fundamental Question.**

  `How to classify curves and surfaces.` [6]
- **348 Plan Revisited.**
  - `Weeks 1--3. Representation by functions.` Curves and surfaces in calculus.
  - `Weeks 4--6. Simple properties from differentiation and integration.` Curves in $\mathbb{R}^2$ and $\mathbb{R}^3$, measurements on surfaces.
  - `Weeks 7--13. Attempts to answer the fundamental question.` Curvatures of surfaces.

[6] Compare with the fundamental issue in Euclidean geometry: congruence.

Beyond Classical Differential Geometry

Modern Geometries

- **Modern Differential Geometry.**
  - Higher dimensions;
  - No ambient Euclidean space.
- **Geometric Measure Theory.**
  - More general shapes than curves and surfaces.
- **Topology.**
  - Global properties.
- **Algebraic Geometry.**
  - Geometry via modern algebra.
- **Discrete Differential Geometry.**
  - Geometry for computers.
- **etc.**

Applications in Science and Technology

- Mechanics.
- Image Processing.
- Computer Graphics.
- Biology.
- And many more.

See you Thursday!

- **Review of Multivariable Calculus.**
  1. Vectors;
  2. Functions;
  3. Differentiation.
- **Review of Linear Algebra.**
  1. Linear transformations and matrices;
  2. Linear dependence/independence.
- **Review of Differential Equations.**
  1. Basic solution techniques;
  2. Basic existence/uniqueness theories.

Welcome to Math 348, and

let's all have a successful semester!