# Homework 8

(Total 20 pts; Due Dec. 1 12pm)

Question 1. (5 pts) *Let $S_1$ be a surface patch parametrized by $\sigma_1(u, v)$. Let $S_2$ be the surface patch parametrized by $\sigma_2(u,v) = 2\,\sigma_1(u,v)$. Prove that the following relation holds for the Gaussian curvatures: $K_2(u,v) = \frac{1}{2}\,K_1(u,v)$.*

Question 2. (5 pts) *Let $S$ be a surface with first fundamental form $3\,\mathrm{d}u^2 + 4\,\mathrm{d}v^2$. Let $\mathbb{L}\,\mathrm{d}u^2 + 2\,\mathbb{M}\,\mathrm{d}u\,\mathrm{d}v + \mathbb{N}\,\mathrm{d}v^2$ be its second fundamental form. Prove that $\mathbb{L}_{vv} = \mathbb{N}_{uu}$.*

Question 3. (10 pts) *Consider a surface with first fundamental form $v\,\mathrm{d}u^2 + u^2\,\mathrm{d}v^2$ (we assume $v > 0$)..*

*a*) (5 pts) *Calculate the Christoffel symbols $\Gamma^k_{ij}$.*

*b*) (5 pts) *Can this surface have second fundamental form $u^{-1}\,\mathrm{d}u\,\mathrm{d}v$? Justify your claim.*