Homework 7

(Total 20 pts; Due Nov. 24 12pm)

Question 1. (10 pts) *Consider the surface patch $\sigma(u,v) = (u, v, 3\,u^2 + 2\,v^2)$. Calculate the Christoffel symbols Γ^k_{ij} at the point $(1, 0, 3)$ by solving the equations $\sigma_{uu} = \Gamma^1_{11}\,\sigma_u + \Gamma^2_{11}\,\sigma_v + \mathbb{L}\,N$, etc.*

Question 2. (5 pts) *Consider a surface with first fundamental form $(1 + 36\,u^2)\,\mathrm{d}u^2 + 48\,u\,v\,\mathrm{d}u\,\mathrm{d}v + (1 + 16\,v^2)\,\mathrm{d}v^2$. Calculate the Christoffel symbols Γ^k_{ij} at $(u, v) = (1, 0)$ through the formulas $\Gamma^1_{11} = \frac{\mathbb{G}\,\mathbb{E}_u - 2\,\mathbb{F}\,\mathbb{F}_u + \mathbb{F}\,\mathbb{E}_v}{2\,(\mathbb{E}\,\mathbb{G} - \mathbb{F}^2)}$, etc.*

Question 3. (5 pts) *Let S be the surface $z = x^2 + y^2$. Let $\mathcal{C}$ be the intersection of S with the plane $z = 1$. Let w be the tangent vector field $(-y, x)$ along $\mathcal{C}$. Is w parallel along $\mathcal{C}$? Justify your claim.*