Homework 6

(Total 20 pts; Due Nov. 3 12pm)

Question 1. (10 pts) *Consider the surface patch* $\sigma(u,v)=(u,2v,uv)$. *Calculate* H, K, $\kappa_1, \kappa_2, t_1, t_2$ *at* $p=(1,2,1)$.

Question 2. (5 pts) *Let* γ *be a curve in a surface* S. *Assume that at every* $p\in\gamma$

i. $\dot{\gamma}(p)$ *is parallel to the principal vector* t_1;

ii. *the angle between the osculating plane and* T_pS *is fixed;*

iii. *the normal curvature* $\kappa_n \neq 0$.

Prove that γ *is a plane curve. (Hint: Prove that* $\dot{N}_S \parallel T$*, then calculate* $\frac{\mathrm{d}}{\mathrm{d}s}(N_S \cdot B)$.*)*

Question 3. (5 pts) *Let* S_1, S_2 *be two surfaces. Let the curve* γ *be their intersection. Let* $p\in\gamma$. *Let the normal curvatures at* p *of* S_i *along* γ *be* $\kappa_n^{(i)}$, $i=1,2$. *Let* θ *be the angle between the surface normals at* p. *Prove that*

$$\kappa^2 \sin^2\theta = \left(\kappa_n^{(1)}\right)^2 + \left(\kappa_n^{(2)}\right)^2 - 2\,\kappa_n^{(1)}\,\kappa_n^{(2)} \cos\theta. \tag{1}$$

(Hint: Prove that $\kappa_n^{(i)} = \kappa\cos\theta_i$*)*