Homework 5

(Total 20 pts; Due Oct. 27 12pm)

QUESTION 1. (5 PTS) *Consider the surface patch $\sigma(u, v) = (u^3 v, u^2 + v^2, v)$. Calculate its second fundamental form at $p = (1, 2, 1)$.*

QUESTION 2. (10 PTS) *Let S be a surface patch.*

a) (5 PTS) *Prove that the normal curvature at $p \in S$ in the direction $w \in T_p S$ is*

$$\kappa_n = \frac{\langle\langle w, w \rangle\rangle_{p,S}}{\langle w, w \rangle_{p,S}} \tag{1}$$

b) *Let the first and second fundamental forms of S be $(1 + v^2)\, du^2 + 2\, u\, v\, du\, dv + (1 + u^2)\, dv^2$ and $\frac{2\, du\, dv}{\sqrt{1 + u^2 + v^2}}$ respectively. Calculate the normal curvature at point $\sigma(1, 1)$ in the direction $\sigma_u + \sigma_v$.*

QUESTION 3. (5 PTS) *Let γ be a curve on a surface S. Let $p \in \gamma$. Assume that the osculating plane of γ at p coincides with $T_p S$. Prove that the normal curvature of S at p in the direction $\dot{\gamma}$ is zero.*