Homework 4

(Total 20 pts; Due Oct. 20 12pm)

Question 1. (5 pts) *Consider the surface patch $\sigma(u, v) = (u^3\, v,\, u^2 + v^2,\, v)$. Calculate its first fundamental form at $p = (1, 2, 1)$.*

Question 2. (10 pts) *Let S be a surface patch with first fundamental form $(1 + v^2)\, du^2 + 2\, u\, v\, du\, dv + (1 + u^2)\, dv^2$. Calculate the following.*

   i. (4 pts) *The arc length of the curve $u = t, v = t$ for $0 \leqslant t \leqslant 1$.*

   ii. (4 pts) *The angle between the curves $u = 1$ and $v = 1$.*

   iii. (2 pts) *The area of $\sigma(U)$ where U is the region bounded by the positive u, v axes and the quarter circle $u^2 + v^2 = 1$.*

Question 3. (5 pts) *Prove that the following is an equiareal mapping from the unit sphere to the plane:*

$$f(\cos u \cos v,\, \cos u \sin v,\, \sin u) = (u,\, v \cos u). \tag{1}$$

This is a mapping projection obtained by Sanson in 1650.