Homework 8: Gauss and Codazzi Equations

(Total 20+5 pts; Due Nov. 25 12pm)

QUESTION 1. (10 PTS) *Decide whether there is a parametrized surface $\sigma(u, v)$ with*

    a) (5 PTS) $\mathbb{E} = \mathbb{G} = 1, \mathbb{F} = 0$ *and* $\mathbb{L} = \mathbb{N} = e^{2u}, \mathbb{M} = 0$.

    b) (5 PTS) $\mathbb{E} = 1, \mathbb{F} = 0, \mathbb{G} = \sin^2 u$, $\mathbb{L} = \sin^2 u, \mathbb{M} = 0, \mathbb{N} = 1$.

QUESTION 2. (10 PTS) *Let S be a surface with first fundamental form $u^2 \, du^2 + \beta \, u^2 \, dv^2$ for some $\beta > 0$, and second fundamental form $A(u, v) \, du^2 + B(u, v) \, dv^2$.*

    a) (5 PTS) *Find β.*

    b) (5 PTS) *Prove that $A(u, v), B(u, v)$ are functions of u only.*

> The following are more abstract or technical questions. They carry bonus points.

QUESTION 3. (5 PTS) *Let S be such that $\kappa_1 \neq \kappa_2$ are both constants. Prove that its Gaussian curvature is 0.*

    (You can assume that the surface patch is such that $\sigma_u \parallel t_1, \sigma_v \parallel t_2$ where t_1, t_2 are the principal vectors.)