Homework 2: Basics of Curves and Surfaces

(Total 20 pts + bonus pts; Due Sept. 23 12pm)

Question 1. (5 pts) *Let $a, b, c > 0$.*

a) (2 pts) *Sketch the curve*

$$x(t) = \left(3\cos\frac{t}{c}, 3\sin\frac{t}{c}, 4\,t\right). \tag{1}$$

b) (3 pts) *Find a value of c such that t is the arc length parameter in (1). Justify your claim.*

Question 2. (5 pts) *Consider the hypoboloid $x_3^2 = x_1^2 + x_2^2 + 2$. Calculate its unit normal vectors at the point $(1, 1, 2)$. Note that there are two unit normal vectors at each point.*

Question 3. (5 pts) *Calculate the surface area of the torus in Exercise 4.2.5 in the textbook.*

Question 4. (5 pts) *Let S be the surface $x_3 = x_1^2 + x_2^2$. Let $\mathbb{S}^2$ denote the unit sphere in $\mathbb{R}^3$. Let $N(p)$ be the unit normal vector at $p \in S$, pointing down.*

a) (1 pts) *Calculate $N(p_0)$ for $p_0 = (0, 0, 0)$.*

b) (2 pts) *Write down a surface patch σ for S covering p_0, and a surface patch $\tilde{\sigma}$ for $\mathbb{S}^2$ covering $N(p_0)$.*

c) (2 pts) *Calculate the matrix representation of the differential $D_{p_0}N : T_{p_0}S \mapsto T_{N(p_0)}\mathbb{S}^2$ with respect to the two surface patches you have just specified.*

The following are more abstract or technical questions. They carry bonus points.

Question 5. (**Bonus, 5 pts**) *Let S be the torus described in Exercise 4.2.5 in the textbook. Find an atlas of surface patches for S. You need to write down explicit formulas, including the domain U, for each patch.*