Math 334 Fall 2011: Summary of Quiz 5

OcMountain Daylight Time. 3, 2011

Solution and Grading Scheme.

- *Problem*: Solve
$$y'' - 2\,y' + y = 3\,e^t. \tag{1}$$
- *Solution*: We use undetermined coefficients.
  First solve the homogeneous equation
$$y'' - 2\,y' + y = 0 \tag{2}$$
  The characteristic equation is
$$r^2 - 2\,r + 1 = 0 \Longrightarrow r_1 = r_2 = 1. \tag{3}$$
  So $y_1 = e^t$, $y_2 = t\,e^t$.
  Next guess the form of y_p. As
$$g = 3\,e^t = e^{\alpha t}\,(a_0 + \cdots + a_n\,t^n) \tag{4}$$
  with $\alpha = 1, n = 0$, we guess
$$y_p = t^s\,e^t\,A_0. \tag{5}$$
  Now as $\alpha = 1$ is a repeated root of the characteristic equation, $s = 2$. So
$$y_p = t^2\,e^t\,A_0. \tag{6}$$
  Substitute into the equation we get
$$A_0 = \frac{3}{2}. \tag{7}$$
  So the solution is given by
$$y = C_1\,e^t + C_2\,t\,e^t + \frac{3}{2}\,t^2\,e^t. \tag{8}$$
- It is also OK to solve using variation of parameters. As $y_1 = e^t$, $y_2 = t\,e^t$, we compute
$$W[y_1, y_2] = (e^t)\,(t\,e^t)' - (e^t)'\,(t\,e^t) = e^{2t}. \tag{9}$$
  Note that the equation is already in standard form so $g = 3\,e^t$. Now
$$u_1 = \int \frac{-g\,y_2}{W} = \int \frac{-3\,e^t\,t\,e^t}{e^{2t}} = \int -3\,t = -\frac{3}{2}\,t^2; \tag{10}$$
$$u_2 = \int \frac{g\,y_1}{W} = \int \frac{3\,e^t\,e^t}{e^{2t}} = \int 3 = 3\,t. \tag{11}$$
  So
$$y_p = u_1\,y_1 + u_2\,y_2 = \frac{3}{2}\,t^2\,e^t. \tag{12}$$
  and solution is given by
$$y = C_1\,e^t + C_2\,t\,e^t + \frac{3}{2}\,t^2\,e^t. \tag{13}$$
- *Grading Scheme*:
  - Know the overall procedure: formulas to use, etc. 2 pts;
  - Solving the homogeneous equation: 1 pt.
  - Getting y_p correctly 1pt.
  - Final answer: 1 pt.

Statistics.

5	4	3	2	1	0	Total
12	5	1	3	0	0	21

Table 1. Grade distribution

Popular Mistakes.

- Wrong characteristic equation.
  $y' = r\,e^{rt}$, $y'' = e^{rt} + r^2\,e^{rt}$. The differentiation is with respect to t, not r. So r is just a "constant here". So $(r\,e^{rt})' = r\,(e^{rt})' = r^2\,e^{rt}$.
- Wrong s in $t^s\,e^{\alpha t}\,(A_0)$.
- Wrong roots:
$$r^2 - 2\,r + 1 = 0, \qquad r_1 = r_2 = -1. \tag{14}$$
- Not careful enough:
$$\ldots - \frac{3}{2}\,t^2\,e^t + 3\,t^2\,e^t \tag{15}$$
cancel.
$$\int 3 = t. \tag{16}$$
- Forgot y_1, y_2.
$$u_1 = \frac{3}{2}\,t^2, u_2 = 3\,t^2, \ldots y = \ldots + \frac{3}{2}\,t^2 + 3\,t^2. \tag{17}$$

Some Suggestions.