

MATH 334 FALL 2011: SUMMARY OF QUIZ 3

SEPTEMBER 23, 2011

Solution and Grading Scheme.

- *Problem:* Solve

$$y' = 2xy + x^3. \quad (1)$$

- Tests:

- Familiarity with linear equations;
- Ability to integrate by parts.

- *Solution:* First notice that this is linear equation. We write it into standard form:

$$y' - 2xy = x^3. \quad (2)$$

So $p(x) = -2x$. The integrating factor is computed as

$$\mu(x) = e^{-\int 2x} = e^{-x^2}. \quad (3)$$

Multiply both sides by this integrating factor we get

$$(e^{-x^2}y)' = e^{-x^2}[y' - 2xy] = e^{-x^2}x^3. \quad (4)$$

We need to integrate $e^{-x^2}x^3$.

$$\begin{aligned} \int x^3 e^{-x^2} dx &= \frac{1}{2} \int x^2 e^{-x^2} dx^2 \\ &\quad \text{(Set } u = x^2\text{)} \\ &= \frac{1}{2} \int u e^{-u} du \\ &= -\frac{1}{2} \int u de^{-u} \\ &= -\frac{1}{2} \left[u e^{-u} - \int e^{-u} du \right] \\ &= -\frac{1}{2} [u e^{-u} + e^{-u}] \\ &\quad \text{(Back to } x\text{)} \\ &= -\frac{1}{2} [x^2 e^{-x^2} + e^{-x^2}]. \end{aligned} \quad (5)$$

Therefore

$$e^{-x^2}y = -\frac{1}{2} [x^2 e^{-x^2} + e^{-x^2}] + C \implies y = -\frac{x^2 + 1}{2} + C e^{x^2}. \quad (6)$$

Remark 1. It's also OK not doing $u = x^2$. Just do

$$\int x^3 e^{-x^2} dx = -\frac{1}{2} \int x^2 de^{-x^2} = -\frac{1}{2} \left[x^2 e^{-x^2} - \int e^{-x^2} dx^2 \right] = -\frac{1}{2} [x^2 e^{-x^2} + e^{-x^2}]. \quad (7)$$

- *Grading Scheme:*

- Know how to solve: 2 pts;
- Correct integrating factor: 1 pt.
- Correct integration of the right hand side: 1 pt;
- Correct final answer: 1 pt.

Statistics.

5	4	3	2	1	0	Total
2	0	5	1	0	1	9

Table 1. Grade distribution**Popular Mistakes.**

- Wrong $p(x)$.

$$p(x) = 2x. \quad (8)$$

- Forget to multiply the right hand side by $\mu(x)$:

$$\mu(x) = e^{-x^2}, \text{ so } (e^{-x^2}y)' = x^3. \quad (9)$$

- Wrong cancellation.

$$(e^{x^2}y)' = x^3 e^{-x^2} \implies y' = x^3 e^{-x^2} e^{-x^2}. \quad (10)$$

- Unable to integrate

$$\int x^3 e^{-x^2} dx. \quad (11)$$

Some Suggestions.

- It would be great if you can hand in your quiz even if you cannot solve the problem. Only then can I get correct estimates of the strength and weakness of the class.