Math 334 Fall 2011: Summary of Quiz 1

September 12, 2011

Solution and Grading Scheme.

- *Problem*: Solve $\dot{y} = t\,e^t$.
- *Solution*: All we need to do is to find the primitive of $t\,e^t$. Calculate:

$$\begin{aligned}\int t\,e^t\,\mathrm{d}t &= \int u\,\mathrm{d}v \qquad (u=t, v=e^t)\\ &= u\,v - \int v\,\mathrm{d}u\\ &= t\,e^t - \int e^t\,\mathrm{d}t \qquad \text{(Integration by parts)}\\ &= t\,e^t - e^t\\ &= e^t\,(t-1). \end{aligned} \tag{1}$$

  For those who are familiar with the integration by parts process, the step $u=t, v=e^t$ can be omitted. Just write

$$\int t\,e^t\,\mathrm{d}t = \int t\,\mathrm{d}e^t = t\,e^t - \int e^t\,\mathrm{d}t = t\,e^t - e^t = (t-1)\,e^t. \tag{2}$$

  Now write down the solution (**don't forget!**)

$$y = (t-1)\,e^t + C. \tag{3}$$

- *Grading Scheme*:
  - Know how to solve: 2pts;
  - Know how to do integration by parts: 1 pt;
  - Evaluate the integral correctly: 1 pt;
  - Correct final answer: 1 pt.

Statistics.

5	4	3	2	1	0	Total
9	8	2	2	0	2	23

Table 1. Grade distribution

Popular Mistakes.

- Forget to include an arbitrary constant C.
- Remember incorrectly the integration by parts formula.

  Integration by parts is the "reverse" of the Leibniz formula for differentiating product of two functions.

$$(f\,g)' = f'\,g + g'\,f \Longrightarrow f\,g = \int f'\,g + \int g'\,f \Longrightarrow \int f'\,g = f\,g - \int g'\,f. \tag{4}$$

  Or use the d operation:

$$\mathrm{d}(f\,g) = g\,\mathrm{d}f + f\,\mathrm{d}g \Longrightarrow f\,g = \int g\,\mathrm{d}f + \int f\,\mathrm{d}g \Longrightarrow \int g\,\mathrm{d}f = f\,g - \int f\,\mathrm{d}g. \tag{5}$$

- Remember incorrectly basic integration facts such as $\int e^t\,\mathrm{d}t = e^t + C$.

Some Suggestions.

- Write more intermediate steps. For example, the following answer

$$\dot{y} = t\,e^t \text{ therefore } y = (t+1)\,e^t$$

would get 0 point, while this one

$$\dot{y} = t\,e^t \Longrightarrow y = \int t\,e^t\,\mathrm{d}t = (t+1)\,e^t$$

would get 2 (because knowledge of how to solve the equation is shown), and finally

$$\dot{y} = t\,e^t \Longrightarrow y = \int t\,e^t\,\mathrm{d}t = t\,e^t - \int e^t\,\mathrm{d}t = (t+1)\,e^t$$

would get 3 (because furthermore the correct procedure of integration by parts is shown).

- It is a good idea to plug your solution back into the equation and check whether it's correct.