Oct. 21, 2011

Math 334 Fall 2011 Homework 7

Basic

Problem 1. Determine the radius of convergence for the following:

a) $\sum_{n=0}^{\infty} n^2 (x-1)^n$.

b) $\sum_{n=1}^{\infty} \frac{(2x+1)^n}{n^2}$.

c) $\sum_{n=0}^{\infty} \frac{n! x^n}{n^n}$.

Problem 2. Calculate the Taylor expansion for the following functions.

a) $\ln x$, $x_0 = 1$;

b) e^{x^3}, $x_0 = 0$;

c) $\frac{1}{1-x}$, $x_0 = 2$.

Problem 3. Rewrie the given expression as a sum whose generic term involves x^n:

a) $\sum_{n=0}^{\infty} a_n x^{n+2}$;

b) $(1-x^2) \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$.

Problem 4. Find the first five nonzero terms in the solution of the problem

$$y'' - x y' - y = 0, \qquad y(0) = 2, \quad y'(0) = 1. \tag{1}$$

Intermediate

Advanced

Problem 5. **Use power series** to solve

$$y'' - 2y' + y = 0. \tag{2}$$

Challenge

Problem 6. Consider two power series $\sum_{n=0}^{\infty} a_n x^n$ and $\sum_{n=0}^{\infty} b_n x^n$ with radii of convergence ρ_1, ρ_2 respectively. We claim that the radius of convergence for the power series $\sum_{n=0}^{\infty} c_n x^n$ corresponding to the ratio $\sum_{n=0}^{\infty} a_n x^n / \sum_{n=0}^{\infty} b_n x^n$ is **at least**

$$\min\left(\text{Distance between 0 and the closest zero of } \sum_{n=0}^{\infty} b_n x^n, \ \rho_1, \ \rho_2\right). \tag{3}$$

Construct an example to show that "at least" cannot be dropped.

See Next Page for Answers

ANSWERS

- Problem 1
  - a) 1
  - b) 1/2
  - c) e
- Problem 2
  - a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} (x-1)^n$.
  - b) $\sum_{n=0}^{\infty} \frac{x^{3n}}{n!}$
  - c) $\sum_{n=0}^{\infty} (-1)^{n+1}(x-2)^n$.
- Problem 3
  - a) $\sum_{n=2}^{\infty} a_{n-2}\, x^n$
  - b) $2\, a_2 + 6\, a_3\, x + \sum_{n=2}^{\infty} [(n+2)\,(n+1)\, a_{n+2} - n\,(n-1)\, a_n]\, x^n$
- Problem 4
  $y(x) = 2 + x + x^2 + \frac{1}{3}\, x^3 + \frac{1}{4}\, x^4 + \cdots$
- Problem 5
  $(a_1 - a_0)\, x\, e^x + a_0\, e^x$.