

MATH 334 FALL 2011 HOMEWORK 4

BASIC

Problem 1. Solve the following equations:

a) $3y'' + 8y' + 4y = 0.$

b) $y'' + 6y' + 9y = 0.$

c) $y'' + 2y' + 10y = 0.$

Problem 2. Solve the following initial value problem.

a) $y'' + 3y' - 4y = 0, y(1) = 0, y'(1) = 1.$

b) $y'' + 2y' + 4y = 0, y(0) = 1, y'(0) = 1.$

INTERMEDIATE

Problem 3. Find the general solution for

$$y'' + 2y' + y = 2e^{-t}. \tag{1}$$

Problem 4. Solve the initial value problem

$$y'' + 4y = t^2 + 3e^t, \quad y(0) = 0, \quad y'(0) = 2. \tag{2}$$

ADVANCED

Problem 5. Consider $ay'' + by' + cy = 0$ with a, b, c constants. Assume that $ar^2 + br + c = 0$ has repeated root $r_1 = r_2$. Thus $y_1 = e^{r_1 t}$. Show that reduction of order always gives $y_2 = t y_1$.

CHALLENGE

Problem 6. Explain why the method of undetermined coefficients is not practical anymore when the coefficients are not constants.

Problem 7. Show that reduction of order always works. That is it always gives a y_2 that is linearly independent of y_1 .

See Next Page for Answers

ANSWERS

- Problem 1.
 - a) $y = C_1 e^{-2t/3} + C_2 e^{-2t}$.
 - b) $y = C_1 e^{-3t} + C_2 t e^{-3t}$.
 - c) $y = C_1 e^{-t} \cos 3t + C_2 e^{-t} \sin 3t$.
- Problem 2.
 - a) $y = -\frac{1}{5} e^{4-4t} + \frac{1}{5} e^{t-1}$.
 - b) $y = e^{-t} \cos \sqrt{3} t + \frac{2}{\sqrt{3}} e^{-t} \sin \sqrt{3} t$.
- Problem 3. $y = C_1 e^{-t} + C_2 t e^{-t} + t^2 e^{-t}$.
- Problem 4. $y = -\frac{19}{40} \cos 2t + \frac{7}{10} \sin 2t - \frac{1}{8} + \frac{t^2}{4} + \frac{3}{5} e^t$.