Sept. 16, 2011

Math 334 Fall 2011 Homework 2

Basic

Problem 1. The d operator. Calculate

a) $\mathrm{d}(\sin x + y)$;

b) $\mathrm{d}\Big(\frac{x\,y}{x^2+y^2}\Big)$;

c) $\mathrm{d}(e^{x\,y})$;

Problem 2. Solve the following exact equations.

a) $(6\,x\,y^2 + 4\,x^3\,y)\,\mathrm{d}x + (6\,x^2\,y + x^4 + e^y)\,\mathrm{d}y = 0$.

b) $\Big(\frac{1}{y}\sin\frac{x}{y} - \frac{y}{x^2}\cos\frac{y}{x} + 1\Big)\,\mathrm{d}x + \Big(\frac{1}{x}\cos\frac{y}{x} - \frac{x}{y^2}\sin\frac{x}{y} + \frac{1}{y^2}\Big)\,\mathrm{d}y = 0$.

Problem 3. Solve the following linear equations.

a) Solve

$$y' = 1 + 3y\tan x. \tag{1}$$

b) Solve

$$y' = 2\,x\,y + x; \qquad y(1) = 1. \tag{2}$$

Problem 4. Solve the following separable equations.

a) Solve

$$y' = -\frac{x\,e^x\,y^3}{y+1} \tag{3}$$

b) Solve

$$y' = (\tan x)\,(\tan y) \tag{4}$$

Problem 5. Are the following equations exact?

a) $3\,(x^2+y^2)\,\mathrm{d}x + x\,(x^2 + 3\,y^2 + 6\,y)\,\mathrm{d}y = 0$.

b) $y\,(2\,x - y + 2)\,\mathrm{d}x + 2\,(x-y)\,\mathrm{d}y = 0$.

Intermediate

Problem 6. Solve the equations from Problem 5.

a) $3\,(x^2+y^2)\,\mathrm{d}x + x\,(x^2 + 3\,y^2 + 6\,y)\,\mathrm{d}y = 0$.

b) $y\,(2\,x - y + 2)\,\mathrm{d}x + 2\,(x-y)\,\mathrm{d}y = 0$.

Advanced

Problem 7. Solve

$$y' + \frac{x}{y} + 2 = 0, \qquad y(0) = 1. \tag{5}$$

Challenge

Problem 8. Consider the general linear 1st order equation

$$y' + p(x)\,y = g(x). \tag{6}$$

Write it as $M\,\mathrm{d}x + N\,\mathrm{d}y = 0$ and show that it is exact only when $p(x) = 0$. Explore possible integrating factors using the general theory.

See Next Page for Answers

ANSWERS

- Problem 1:
  a) $\cos x \, \mathrm{d}x + \mathrm{d}y$;
  b) $\frac{y\,(y^2 - x^2)}{(x^2 + y^2)^2} \, \mathrm{d}x + \frac{x\,(x^2 - y^2)}{(x^2 + y^2)^2} \, \mathrm{d}y$;
  c) $y\, e^{xy} \, \mathrm{d}x + x\, e^{xy} \, \mathrm{d}y$.
- Problem 2:
  a) $3\, x^2\, y^2 + x^4\, y + e^y = C$.
  b) $-\cos \frac{x}{y} + x - \frac{1}{y} + \sin \frac{y}{x} = C$.
- Problem 3:
  a) $\frac{1}{\cos^3 x} \Big(\sin x - \frac{1}{3} \sin^3 x + C \Big)$.
  b) $\frac{3}{2\,e}\, e^{x^2} - \frac{1}{2}$.
- Problem 4:
  a) $(x - 1)\, e^x - \Big(y^{-1} + \frac{1}{2}\, y^{-2} \Big) = C$ and $y = 0$;
  b) $(\sin y)\, (\cos x) = C$.
- Problem 5:
  a) No.
  b) No.
- Problem 6:
  a) $x^3\, e^y + 3\, x\, y^2\, e^y = C$;
  b) $2\, e^x\, x\, y - e^x\, y^2 = C$.
- Problem 7: $\ln |y + x| + \frac{x}{y + x} = 0$.