MATH 334 FALL 2011 HOMEWORK 1

Basic

Problem 1. Go to http://www.math.rutgers.edu/~sontag/JODE/JOdeApplet.html, plot the slope fields of the following equations, and then imagine what the integral curves should look like.

- a) y' = 3x 5y;
- b) $\dot{x} = (x 2t)(x + t);$
- c) $\frac{dy}{dx} = \ln|x y|$ (type "ln(abs(x-y))")

Problem 2. Check solutions.

a) $y = C_1 e^{-2x} + C_2 e^x + \sin 3x$ solves

$$y'' + y' - 2y = -11\sin 3x + 3\cos 3x. \tag{1}$$

b) $y = x^3$ solves

$$x^2y'' - xy' - 3y = 0. (2)$$

Problem 3. Solve the following differential equations.

- a) $\frac{\mathrm{d}y}{\mathrm{d}x} = e^x \sin x;$
- b) $\dot{y} = t \sin t$;
- c) $3y^2y' = x^2$.

Intermediate

Problem 4. Find the values of α such that $e^{\alpha x}$ solves

$$y'' + 2y' + 4y = 0. (3)$$

Problem 5. Find the values of r such that x^r solves

$$x^{2}y'' + 6xy' + 4y = 0. (4)$$

Advanced

Challenge

See next page for answers.

Answers

Problem 3:

a)
$$y = \frac{1}{2} (e^x \sin x - e^x \cos x) + C$$
.

b)
$$y = -t\cos t + \sin t + C$$
.

c)
$$y^3 = \frac{1}{3}x^3 + C$$
.

Problem 4:
$$-1 \pm \sqrt{3} i$$
.
Problem 5: $-4, -1$.

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