

MATH 334 A1 HOMEWORK 5 (DUE DEC. 8 5PM)

- No “Advanced” or “Challenge” problems will appear in homeworks.

BASIC PROBLEMS

Problem 1. (6.3 1) Sketch for $t \geq 0$

$$g(t) = u(t-1) + 2u(t-3) - 6u(t-4). \quad (1)$$

Problem 2. (6.3 12) Express

$$f(t) = \begin{cases} t & 0 \leq t < 2 \\ 2 & 2 \leq t < 5 \\ 7-t & 5 \leq t < 7 \\ 0 & t \geq 7 \end{cases} \quad (2)$$

in terms of the unit step function.

Problem 3. (6.3 13) Find the Laplace transform for

$$f(t) = \begin{cases} 0 & t < 2 \\ (t-2)^2 & t \geq 2 \end{cases}. \quad (3)$$

Problem 4. (6.3 21) Find the inverse Laplace transform of

$$F(s) = \frac{2(s-1)e^{-2s}}{s^2 - 2s + 2}. \quad (4)$$

Problem 5. (6.3 24) Find the inverse Laplace transform of

$$F(s) = \frac{e^{-s} + e^{-2s} - e^{-3s} - e^{-4s}}{s}. \quad (5)$$

INTERMEDIATE PROBLEMS

Problem 6. (6.3 33) Find the Laplace transform of

$$f(t) = 1 + \sum_{k=1}^{\infty} (-1)^k u(t-k). \quad (6)$$

(You can assume that term-by-term integration is permissible)

Problem 7. (6.4 9) Solve

$$y'' + y = g(t) = \begin{cases} t/2 & 0 \leq t < 6 \\ 3 & t \geq 6 \end{cases}, \quad y(0) = 0, \quad y'(0) = 1. \quad (7)$$

Problem 8. (6.5 10) Solve

$$2y'' + y' + 4y = \delta(t - \pi/6) \sin t; \quad y(0) = 0, \quad y'(0) = 0. \quad (8)$$

Problem 9. (6.6 4) Find the Laplace transform of

$$f(t) = \int_0^t (t-\tau)^2 \cos 2\tau \, d\tau. \quad (9)$$

Problem 10. (6.6 8) Find the inverse Laplace transform using the convolution theorem

$$F(s) = \frac{1}{s^4(s^2+1)}. \quad (10)$$

Problem 11. (7.1 6) Transform the given initial value problem into an initial value problem of two first order equations:

$$u'' + p(t)u' + q(t)u = g(t), \quad u(0) = u_0, \quad u'(0) = u'_0. \quad (11)$$