Math 334 A1 Homework 4 (Due Nov. 26 5pm)

- No "Advanced" or "Challenge" problems will appear in homeworks.

Basic Problems

Problem 1. (6.1.9) Find the Laplace transform of

$$f(t) = e^{at}\cosh bt \tag{1}$$

where $\cosh bt$ is defiend as $\left(e^{bt}+e^{-bt}\right)/2$.

Problem 2. (6.2 1) Find the inverse Laplace transform of

$$F(s) = \frac{3}{s^2+4}. \tag{2}$$

Problem 3. (6.2 5) Find the inverse Laplace transform of

$$F(s) = \frac{2s+2}{s^2+2s+5}. \tag{3}$$

Problem 4. (6.2 8) Find the inverse Laplace transform of

$$F(s) = \frac{8s^2-4s+12}{s(s^2+4)}. \tag{4}$$

Problem 5. (6.2 12) Use Laplace transform to solve

$$y''+3y'+2y=0; \qquad y(0)=1, \quad y'(0)=0. \tag{5}$$

Problem 6. (6.2 17) Use Laplace transform to solve

$$y^{(4)}-4y'''+6y''-4y'+y=0; \qquad y(0)=0, \quad y'(0)=1, \quad y''(0)=0, \quad y'''(0)=1. \tag{6}$$

Intermediate Problems

Problem 7. (5.5 8) Consider

$$2x^2y''+3xy'+\left(2x^2-1\right)y=0. \tag{7}$$

a) Show that the equation has a regular singular point at $x=0$;

b) Determine the indicial equation, the recurrence relation, and the roots of the indicial equation;

c) Find the series solution ($x>0$) corresponding to the larger root;

d) If the roots are unequal and do not differ by an integer, find the series solution corresponding to the smaller root also.

Problem 8. (6.1 22) Determine whether

$$\int_0^\infty te^{-t}\,\mathrm{d}t \tag{8}$$

converges or diverges.