NAME:

MATH 317 Q1 WINTER 2017 QUIZ 3

Feb. 17, 2017, 25 minutes

• The quiz has 3 problems. Total 10 + 1 points.

QUESTION 1. (5 PTS) Calculate the surface area of the torus

$$((a+b\cos\theta)\cos\varphi, (a+b\cos\theta)\sin\varphi, b\sin\theta)$$
(1)

where 0 < a < b.

QUESTION 2. (5 PTS) Let $f(x, y, z) := \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and S be the portion of $x^2 + y^2 = z$ with $1 \leq z \leq 4$, oriented so that the normal points upward. Calculate $\int_S f \cdot dS$

QUESTION 3. (1 BONUS PT) Let $\Omega \subset \mathbb{R}^3$ be the region bounded by x = 0, y = 0, z = 0 and x + y + z = 1. Let $\partial \Omega$ be its boundary. Let $f, g, h \in C^1$. Prove

$$\int_{\partial\Omega} f \,\mathrm{d}y \wedge \mathrm{d}z + g \,\mathrm{d}z \wedge \mathrm{d}x + h \,\mathrm{d}x \wedge \mathrm{d}y = \int_{\Omega} \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z} \right) \mathrm{d}x \,\mathrm{d}y \,\mathrm{d}z, \qquad (2)$$

where the normal of $\partial \Omega$ points outward.

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