An Over-simplified History of Calculus

A) When do counting and measuring start?

B) Counting $\Longrightarrow$ Number theory;

C) Measuring $\Longrightarrow$ Geometry;

D) The first attempt to unify failed: There are geometric quantities such as $\sqrt{2}$ that cannot be included in number theory.

E) Ancient Greek solution: Do algebra through geometry.

Example 1. Let two line segments be given. Denote their lengths by a, b. Find another line segment whose length is $c = \sqrt{a\,b}$.
Solution.

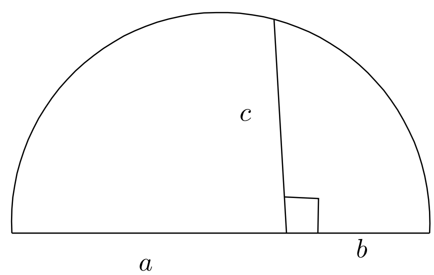

Figure 1. Finding $\sqrt{a\,b}$ geometrically.

Exercise 1. Can we find $\sqrt{a\,b\,c}$ for given line segments with length a, b, c respectively?

F) Archimedes' calculus: geometric.

G) Other civilizations: Develop theory for solving algebraic equations.

H) Most algebraic equations cannot be solved (Ruffini, Abel, Galois).

I) Key idea of algebra: Manipulate symbols following the same rules for numbers.

J) The idea of a generic function: Nicole Oresme (1320–1384).

K) Second attempt to unify number with geometry: Apply algebra to geometry. Fermat and Descartes.

L) Finding maximum/minimum of polynomials $\Longrightarrow$ primitive calculus-type techniques. Barron, Fermat, etc.

M) Organized theory of calculus: Newton, Leibniz. Generalization of polynomials to infinite degree (power series).

N) 18th century: Application of calculus to everything in mathematics, science, and engineering.

O) A new type of equations: Differential equations.

P) Solving ordinary differential equations $\Longrightarrow$ techniques of indefinite integration.

Q) Indefinite integration has severe limits: Chebyshev's theorem. Few differential equations can be solved explicitly.

R) Solving partial differential equations with power series $\Longrightarrow$ Cauchy-Kowalevskaya theorem.

S) 19th century breakthrough: Fourier series. Very effective in understanding partial differential equations, but without any mathematical justification.

T) Convergence of Fourier series is very subtle. Study of this problem leads to

U) Generalization of the idea of functions;

V) Thorough study of continuity/differentiability of functions;

W) Measure theory: How to measure (by covering) the sizes of the set of those points where certain Fourier series converges;

X) Measure theory $\Longleftrightarrow$ the theory of definite integrals.

Y) Set theory: How to measure (by counting) the sizes of the set of those points where certain Fourier series converges;

Z) Further developments of calculus have their own names: Measure theory, real analysis, functional analysis, etc.