Math 317 Winter 2017 Homework 5

DUE THURSDAY APR. 6, 2017 5PM

- The total points of this homework is 20.

- You need to fully justify your answer – for example, prove that your function indeed has the specified property – for each problem.

QUESTION 1. (8 PTS) *Calculate the explicit formulas of*

a) (4 PTS) $f(x) := \sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}$,

b) (4 PTS) $f(x) := \sum_{n=1}^{\infty} \frac{x^n}{n(n+1)}$.

Justify your calculation.

QUESTION 2. (8 PTS) *Let* $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x & 0 \leqslant x \leqslant \pi \end{cases}$ *and be* 2π-*periodic.*

a) (4 PTS) *Calculate the Fourier series expansion of* $f(x)$;

b) (4 PTS) *Use this expansion to prove*

$$\sum_{n \text{ odd}} \frac{1}{n^2} = \frac{\pi^2}{8}. \tag{1}$$

QUESTION 3. (4 PTS) *Let* $D_N(t)$ *be the Dirichlet kernel. Prove*

$$\lim_{N \to \infty} \int_{-\pi}^{\pi} |D_N(t)| \, \mathrm{d}t = \infty. \tag{2}$$