Math 317 Winter 2017 Homework 3

Due Thursday Mar. 9, 2017 5pm

- This homework consists of 5 problems of 4 points each. The total is 20.
- You need to fully justify your answer – for example, prove that your function indeed has the specified property – for each problem.

Question 1. (4 pts) *Prove that* $\sum_{n=1}^{\infty} n^3 e^{-n}$ *converges.*

Question 2. (4 pts) *Find all values of* $p > 0$ *such that* $\sum_{n=1}^{\infty} p^n n^p$ *is convergent. Justify your claim.*

Question 3. (4 pts) *Prove that if* $\sum_{n=1}^{\infty} a_n$ *converges, then for every fixed* $k \in \mathbb{N}$*, there holds*

$$\lim_{n \to \infty} (a_n + \cdots + a_{n+k}) = 0. \tag{1}$$

Then find a divergent series $\sum_{n=1}^{\infty} a_n$ *such that for every fixed* $k \in \mathbb{N}$, $\lim_{n \to \infty} (a_n + \cdots + a_{n+k}) = 0$.

Question 4. (4 pts) *Assume that* $\sum_{n=1}^{\infty} a_n^2$ *and* $\sum_{n=1}^{\infty} b_n^2$ *converge. Prove that* $\sum_{n=1}^{\infty} \frac{(n+1)^2 a_n b_n}{n^2}$ *converges.*

Question 5. (4 pts) *Prove or disprove:* $a_n > 0$, $a_n \longrightarrow 0$, *then* $\sum_{n=0}^{\infty} (-1)^n a_n$ *converges.*