Math 317 Winter 2017 Homework 1

Due Thursday Feb. 2, 2017 5pm

- This homework consists of 5 problems of 4 points each. The total is 20.
- You need to fully justify your answer – for example, prove that your function indeed has the specified property – for each problem.
- This homework covers material up to and including Jan. 26 lecture.

Question 1. (4 pts) *Consider the function $T: \mathbb{R}^2 \mapsto \mathbb{R}^2$ defined through*

$$T(x, y) = \begin{pmatrix} x^2 - y^2 \\ 2\,x\,y \end{pmatrix}. \tag{1}$$

Show that T does not have an inverse function on any open set $U \ni (0,0)$.

Question 2. (4 pts) *Consider the function $T: \mathbb{R}^2 \mapsto \mathbb{R}^2$ defined through*

$$T(x, y) = \begin{pmatrix} x^2 - y^2 \\ 2\,x\,y \end{pmatrix}. \tag{2}$$

We accept that T has an inverse function near $x = 1$, $y = 1$. Calculate the Jacobian matrix of the inverse function at the corresponding point.

Question 3. (4 pts) *Let an implicit function $Y(x)$ be defined through*

$$y^3 + x^2\,y^2 - x\,y + x^4 = 0 \tag{3}$$

Calculate $Y'(0), Y''(0), Y'''(0)$.

Question 4. (4 pts) *Let $f: \mathbb{R}^3 \mapsto \mathbb{R}$ be a C^1 function. Consider the equation $f(x, y, z) = 0$. Assume that $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$ are all nonzero at (x_0, y_0, z_0) and thus around (x_0, y_0, z_0) we can define x as an implicit function of y, z, y as an implicit function of x, z, and z as an implicit function of x, y. Prove that $\frac{\partial x}{\partial y}\frac{\partial y}{\partial z}\frac{\partial z}{\partial x} = -1$.*

Question 5. (4 pts) *Find the maximum value of $\left|\sum_{k=1}^N a_k\,x_k\right|$ subject to $\sum_{k=1}^N x_k^2 = 1$ using the Lagrange multiplier method.*