<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
1. (30 points) Answer the following questions.

(a) (5 points) Evaluate $e^{\pi i/2}$.

(b) (10 points) Convert the ODE $x''' + x'' + x' + x = 1$ into an equivalent system of three 1st order ODEs and find all the equilibrium points (fixed points) of the system.

(c) (15 points) Suppose that $te^{2000t}$ is a solution of the ODE

$$x'' + ax' + bx = 0$$

where $a$ and $b$ are real constants. What is the general solution of the equation? What are the values of $a$ and $b$?
2. (40 points) Solve the following ODEs.

(a) (10 points) Solve \( x' + tx = t \) with \( x(0) = 2 \).
(b) (10 points) Solve $x'' - 4x' + 4x = 1$. 
(c) (10 points) Solve $x'' + x = 0$ with $x(0) = 1$ and $x'(0) = -1$.
Express your final answer in the phase-amplitude form.
(d) (10 points) Solve $x'' + x = \cos t$ with $x(0) = x'(0) = 0$. 
3. (10 points) For each nonhomogeneous linear ODE in the following list, determine a suitable guess for a trial solution, but do not evaluate the coefficients.

(a) \( x'' - 2x' + x = t^3 e^t + t^3 \)

(b) \( x'' + 2x' + 5x = te^{-t} \cos 2t \)
4. (20 points) Let $a$ and $b$ be real constants. Consider the ODE

$$x'' + ax' + bx = 0.$$ 

(a) For what values of $a$ and $b$ does the corresponding 1st order system have a center at the origin?

(b) For what values of $a$ and $b$ does the corresponding 1st order system have a saddle at the origin?

(c) For what values of $a$ and $b$ does the corresponding 1st order system have a sink at the origin?

(d) For what values of $a$ and $b$ does the corresponding 1st order system have a source at the origin?