| Math 5A Midterm I | Apr 26, 2000 |
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PRINT NAME:_____

PERM NUMBER:_____

DISCUSSION SECTION AND TA'S NAME:_____

| Problem | Points | Score |
|---|---|---|
| 1 | 30 | |
| 2 | 20 | |
| 3 | 20 | |
| 4 | 10 | |
| 5 | 20 | |
| Total | 100 | |

1. (30 points) Answer the following questions.

   (a) (5 points) What is the real part of the complex number $e^{1+i}$?

   (b) (5 points) Convert the ODE $x''' + (x'')^2 + (x')^2 + x^2 = 0$ into an equivalent system of three 1st order ODEs.

   (c) (10 points) Write the function $\cos t + \sin t$ in the phase-amplitude form, i.e., determine the values of $A$ and $\phi$ such that $\cos t + \sin t = A\cos(t - \phi)$. Find its maximum and minimum for $0 \leq t \leq 2\pi$.

(d) (5 points) Find all the equilibrium points (fixed points) of the following system.

$$\begin{cases} x_1' = x_1^2 + x_2^2 - 2 \\ x_2' = x_1 + x_2 \end{cases}$$

(e) (5 points) Suppose that $e^{1999t}\cos(2000t)$ is a solution of the ODE

$$x'' + ax' + bx = 0$$

where $a$ and $b$ are real constants. What is the general solution of the equation?

2. (20 points) Let $f(t)$ and $g(t)$ be two continuous functions on $(0, \infty)$. Suppose that the ODE

$$\frac{d^2x}{dt^2} + f(t)\frac{dx}{dt} + g(t)x = 0$$

has two solutions $t^2$ and $t^3$.

(a) Find the general solution of the equation.

(b) Find the solution of the equation satisfying $x(1) = x'(1) = 1$.

3. (20 points) Solve the following ODEs.

   (a) $tx' + x = t$ with $x(2) = 2$.

(b) $x'' - 4x' + 8x = e^t$

4. (10 points) For each nonhomogeneous linear ODE in the following list, determine a suitable guess for a trial solution, but do not evaluate the coefficients.

   (a) $x'' + 2x' + x = t^3 e^t + t^2$

   (b) $x'' + 2x' + 5x = e^{-t}(\cos 2t + \sin 2t)$

5. (20 points) Let $c$ be a real constant. Consider the ODE

$$x'' + 2cx' + (c+2)x = 0.$$

(a) For what values of $c$ does the corresponding 1st order system have a center at the origin?

(b) For what values of $c$ does the corresponding 1st order system have a saddle at the origin?

(c) For what values of $c$ does the corresponding 1st order system have a sink at the origin?

(d) For what values of $c$ does the corresponding 1st order system have a source at the origin?