

1. Convert the following differential equation into an equivalent system of first-order differential equations.

$$x''' + (x')^2 + x(x - 1) = 0.$$

2. Consider the following system of differential equations

$$\begin{cases} x' = x^2 + y^2 - 2 \\ y' = x^2 - y^2 \end{cases}$$

- (a) Find all the fixed points.
 - (b) Linearize the system about each fixed point.
 - (c) Determine the stability of each fixed point.
3. Let $x'' + cx' + 4x = 0$, where c is a real constant.
 - (a) Determine the type of the fixed point (sink, source, saddle or center) at the origin for $c = -4, 0, 4$, respectively.
 - (b) Is it possible that the origin is a saddle for some value of c ? You must justify your answer.
 4. Consider the following system of differential equations

$$\begin{cases} x' = (1 - x - y/2)x \\ y' = (1 - y - x/2)y \end{cases}$$

- (a) Identify the nullclines where $x' = 0$.
- (b) Identify the nullclines where $y' = 0$.
- (c) Sketch the four regions divided by the nullclines in the first quadrant. Label the four regions as $(+, +), (+, -), (-, +), (-, -)$ according to the signs of x' and y' .
- (d) Find the four fixed points.
- (e) Determine the stability of each fixed point.