Math 5A **Common Final** **Spring 2000**

June 12, 00

Name ________________________

	1	2	3	4	5	6	7	8	Total
Score									

Write out a complete solution to each problem and circle the correct answer. Be sure to show all the details of your method of solution. No credit can be given for solutions with an incomplete method. Partial credit will be given where appropriate.

1. (14 points) The following the initial value problem models an idealized mass-spring system.

$$x'' + 2x' + 2x = 0, \quad x(0) = 1,\ x'(0) = 0.$$

a). Find the solution. Write your solution in the phase-amplitude form.
b). Determine how often the mass crosses the equilibrium position.
c). Find the velocity of the mass when it first crosses the equilibrium position.

2. (10 points) Is [2,7,1] a linear combination of [-1,3,2], [4,1,-1] and [3,17,8]? Prove your answer.

3. (10 points) Solve the following system of linear differential equations with the initial condition $x_1(0) = 3, x_2(0) = 7$.

$$\begin{aligned} \frac{dx_1}{dt} &= 2x_1 + x_2 \\ \frac{dx_2}{dt} &= -x_1 + 2x_2 \end{aligned}$$

4. (14 points) For the following equation

$$x'' + 2x' + 10x = 5 + \cos 2t,$$

a). find the homogeneous part of the solution $x_h(t)$.
b). what is $\lim_{t\to+\infty} x_h(t)$?
c). find the steady state solution.

5. (12 points) True or False:

.... a) $\{[1,3,5],[2,5,8]\}$ is linearly dependent.

.... b) $\det \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = 0.$

.... c) **AB**=**BA** if **A** and **B** are diagonal 2×2 matrices.

.... d) Let **A** and **B** be two 2×2 matrices. If det **AB**=0 then det **A**=0 or det **B**=0.

.... e) Let **A** and **B** be two 2×2 matrices. If **AB**=**0** then **A**=**0** or **B**=**0**.

.... f) $[1,2]^T$ is an eigenvector of $\begin{bmatrix} -4 & 1 \\ 0 & -2 \end{bmatrix}$.

6. (10 points) Convert the following differential equation into an equivalent system of first-order differential equations.

$$x''' + (x')^2 + x(x-1) = 0.$$

7. (15 points) Consider the following system of differential equations

$$\begin{cases} x' &= x^2 + y^2 - 2 \\ y' &= x^2 - y^2 \end{cases}$$

a) Find all the fixed points.
b) Linearize the system about each fixed point.
c) Determine the stability of each fixed point.

8. (15 points) a) Determine the fundamental matrix of the matrix A.

$$A = \begin{pmatrix} 1 & -2 \\ 1 & 4 \end{pmatrix}$$

b) Solve the following system of linear differential equations with the initial condition $x_1(0) = 4, x_(0) = 2$.

$$\begin{aligned} \frac{dx_1}{dt} &= x_1 - 2x_2 + e^t \\ \frac{dx_2}{dt} &= x_1 + 4x_2 - e^t \end{aligned}$$