Math 530 Midterm (Due Feb 28)

(1) Show that $R^{n+1} - \{1 \text{ pt}\}$ and S^n are homotopic.
(2) Compute $H^*_{DR}(\mathbb{R}^2 - \{P, Q\})$, where P and Q are two disctinct points in $\mathbb{R}^2$. Find the closed forms that represent the cohomology classes.
(3) Let M and N be two differential manifolds. Show that M and N are orientable if and only if $M \times N$ is.
(4) Let (A^*, d) and (B^*, d) be two differential complexes. We call two chain maps $f, g : A^* \to B^*$ are homotopic to each other ($f \sim_{hom} g$) if there exists a linear map $K : A^* \to B^{*-1}$ such that $f - g = dK + Kd$. Show the following
   (a) If $f \sim_{hom} g$ and $g \sim_{hom} h$, then $f \sim_{hom} h$, where h is a chain map $A^* \to B^*$.
   (b) If $f \sim_{hom} g$, the $h \circ f \sim_{hom} h \circ g$, where h is a chain map $B^* \to C^*$.
   (c) Let $f^\#$ and $g^\#$ be the maps on the cohomologies $H^*(A^*) \to H^*(B^*)$ induced by f and g, respectively. If $f \sim_{hom} g$, then $f^\# = g^\#$.
(5) Show that the complex project space $\mathbb{CP}^n$ is orientable for all n.