XI CHEN	Math 530 Final	Apr 20, 2007

PRINT NAME:____

(1) Let E and F be two vector bundles over a differential manifold M. We say E is a sub bundle of F if $E \subset F$ and the inclusion $E \hookrightarrow F$ is an injective bundle map. Show that if E is a sub bundle of F, then there exists a sub bundle G of F such that $F = E \oplus G$.

(2) Let E and F be two vector bundles over a differential manifold M. Show that $E \oplus F$ is orientable if both E and F are orientable. Give an example to show that the converse is false.

(3) Give an example of two homotopic but not diffeomorphic differential manifolds M and N of the same dimension.

(4) Let X be a compact orientable differential manifold and $\Delta_X = \{(x,x) : x \in X\}$ be the diagonal of the self product $X \times X$. Let $[\Delta_X] \in H^n(X \times X)$ be the Poincare dual of Δ_X. Then

(a) For any $\omega_1 \in H^k(X)$ and $\omega_2 \in H^l(X)$ with $k + l = n$

$$\int_{X \times X} \pi_1^*\omega_1 \wedge \pi_2^*\omega_2 \wedge [\Delta_X] = \int_X \omega_1 \wedge \omega_2$$

where π_1 and π_2 are the projections $X \times X$ to the two factors.

(b) Find $[\Delta_X]$ for $X = S^1 \times S^1$.