XI CHEN	Math 506 Midterm	March 6, 2018

PRINT NAME:______________________________

STUDENT ID NUMBER:______________________________

(1) No books, notes or calculators are allowed.
(2) Show your work in details.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

(1) (20 points) Let G be a connected open set in $\mathbb{C}$ and $H(G)$ be the space of holomorphic functions on G. For a sequence $\{f_n\} \subset H(G)$ of one-to-one functions which converge to some $f \in H(G)$ locally uniformly, show that f is either one-to-one or a constant function.

(2) (20 points) Find the automorphism group $\mathrm{Aut}(\mathbb{C}^*)$ of biholomorphic maps $f : \mathbb{C}^* \to \mathbb{C}^*$ for $\mathbb{C}^* = \mathbb{C}\backslash\{0\}$.

(3) (20 points) We call a map $f : X \to Y$ *proper* if $f^{-1}(K)$ is compact for all compact sets $K \subset Y$. Show that an entire function $f : \mathbb{C} \to \mathbb{C}$ is proper if and only if $f(z)$ is a nonconstant polynomial in z.

(4) (20 points) Let D be a connected open set in $\mathbb{C}$ and let $H(D)$ be the ring of analytic functions on D. Show that $H(D)$ is integrally closed. Recall that an integral domain R is integrally closed if $f/g \in R$ for all $f, g \neq 0 \in R$ satisfying

$$\left(\frac{f}{g}\right)^n + a_1 \left(\frac{f}{g}\right)^{n-1} + ... + a_n = 0$$

for some $n > 0$ and $a_1, a_2, ..., a_n \in R$.

(5) (20 points) Compute

$$\int_0^\infty \frac{dx}{1 + x^{2018}}$$