Math 506 Final

Apr. 21, 2010

(1) Let $\Lambda_i = \{mu_i + nv_i : m, n \in \mathbb{Z}\} \subset \mathbb{C}$ be two nondegenerated lattices in $\mathbb{C}$ for $i = 1, 2$. Let $\varphi : \mathbb{C}/\Lambda_1 \to \mathbb{C}/\Lambda_2$ be a nonconstant holomorphic map between two complex tori $\mathbb{C}/\Lambda_1$ and $\mathbb{C}/\Lambda_2$. Show that

(a) φ can be lifted to a holomorphic map on their universal coverings, i.e., we have the commutative diagram:

$$\begin{array}{ccc} \mathbb{C} & \xrightarrow{\phi} & \mathbb{C} \\ \downarrow & & \downarrow \\ \mathbb{C}/\Lambda_1 & \xrightarrow{\varphi} & \mathbb{C}/\Lambda_2 \end{array}$$

(b) ϕ in the above diagram is a biholomorphic map $\mathbb{C} \to \mathbb{C}$.

(2) Let f be a holomorphic function on $\mathbb{C}^n$. If there are positive constants C and λ such that

$$|f(z_1, z_2, ..., z_n)| \le C(|z_1|^\lambda + |z_2|^\lambda + ... + |z_n|^\lambda)$$

for all $(z_1, z_2, ..., z_n) \in \mathbb{C}^n$, then $f(z_1, z_2, ..., z_n)$ is a polynomial in $z_1, z_2, ..., z_n$ of degree $\le \lambda$.

(3) Let f be a holomorphic function on $\mathbb{C}^n$. If $f(\mathbb{C}^n)$ misses at least two values, then f must be a constant.

(4) Let $f : \mathbb{C} \to \mathbb{C}/\Lambda$ be a nonconstant holomorphic map from $\mathbb{C}$ to a complex torus $\mathbb{C}/\Lambda$. Show that f is surjective.

(5) Let $f \in \mathcal{O}_2[[z, w]]$ where $f \not\equiv 0$, f is irreducible and $f(0, 0) = 0$. Then $\mathcal{O}_2[[z, w]]/(f) \cong \mathcal{O}_1$ if and only if $\mathcal{O}_2[[z, w]]/(f)$ is integrally closed.